Chapter -9

Mechanical properties of Solids

Inter Molecular force

In a Solid, atoms and Molecules are arranged in such a way that each molecule is acted upon by the forces due to neighbouring molecules. These forces are known as inter molecular forces.

Elasticity

The property of the body to regain its original Configuration (length, Volume or Shape) when the deforming forces are removed, is called elasticity.

The Change in the shape or Size of a body when external forces act on it is determined by the forces between its atoms or molecules. These Short range atomic forces are called "elastic forces".

Perfectly elastic body

A body which regains its original Configur-ation immediatily. and Compditily after the removal of deforming force from it, is called perfectly elastic body. Quartz and phospher bronze are the examples of nearly perfectly elastic bodies.

Plasticity

The inability of a body to return to its original size and shape even on removal of the deforming force is called plasticity and Such a body is called a plastic body.

Stress and Strain

Stress

Stress is defined as the ratio of the internal force F, produced when the

Substances is deformed, to the area A over which this force acts. In equilibrium, this force is equal in magnitude to the externally applied force. In over words

$$\boxed{\text{Stress} = \frac{F}{A}}$$

The SI unit of stress is newton per square metre (Nm^{-2}).

In CGS Units, stress is measured in dyne cm^{-2}.

Dimensional formula of stress is $[ML^{-1}T^{-2}]$.

Stress is of two types -

(i) **Normal Stress** - It is defined as the restoring force per unit area perpendicular to the surface of the body. Normal stress is of two types :

(1) Tensile Stress

(2) Compressive stress

(ii) **Tangential Stress** - When the elastic restoring force or deforming force acts parallel to the surface area, the stress is called tangential stress.

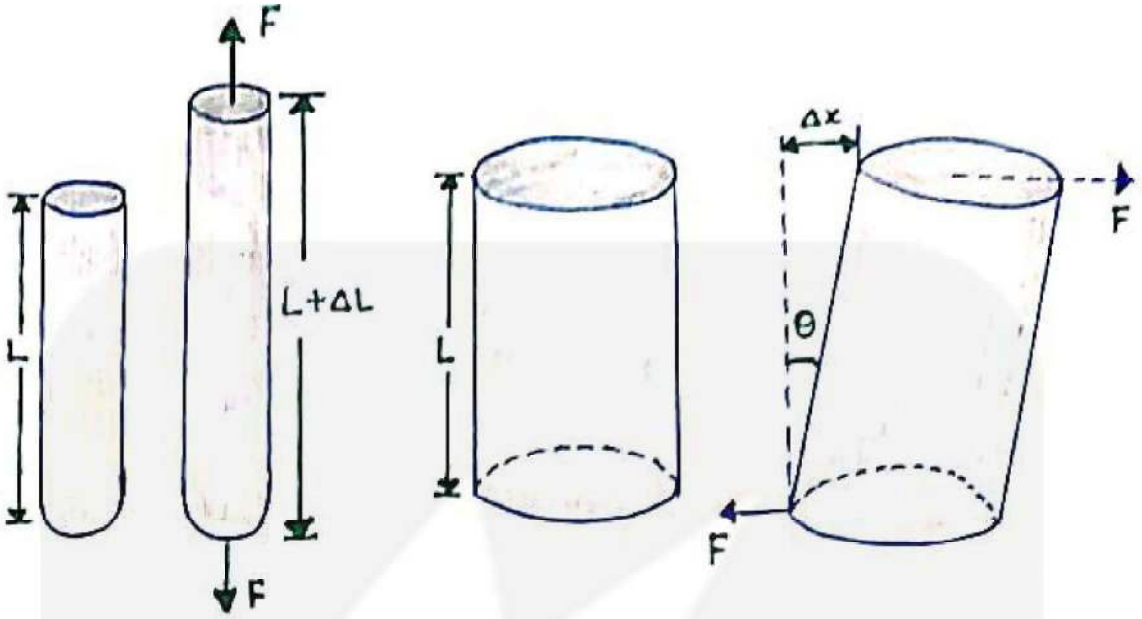
Strain

It is defined as the ratio of the change in size or shape to the original size or shape. It has no dimensions, it is just a number.

Strain is of three types —

(i) **Longitudinal Strain** - If the deforming force produces a change in length alone, the strain produced in the body is called longitudinal strain or tensile strain. It is given as :

$$\boxed{\text{Longitudinal Strain} = \frac{\text{Change in length } (\Delta l)}{\text{Original length } (l)}}$$



(ii) Volumetric strain - If the deforming force produces a change in volume alone, the strain produced in the body is called Volumetric strain.

$$\text{Volumetric Strain} = \frac{\text{Change in Volume}\ (\Delta V)}{\text{Original Volume}\ (V)}$$

(iii) Shear Strain - The angle tilt caused in the body due to tangential stress expressed is called Shear Strain.

$$\boxed{\text{Shear strain} \Rightarrow \theta = \frac{\Delta L}{L}}$$

The maximum stress to which the body can regain its original status on the removal of the deforming force is called elastic limit.

Hooke's Law

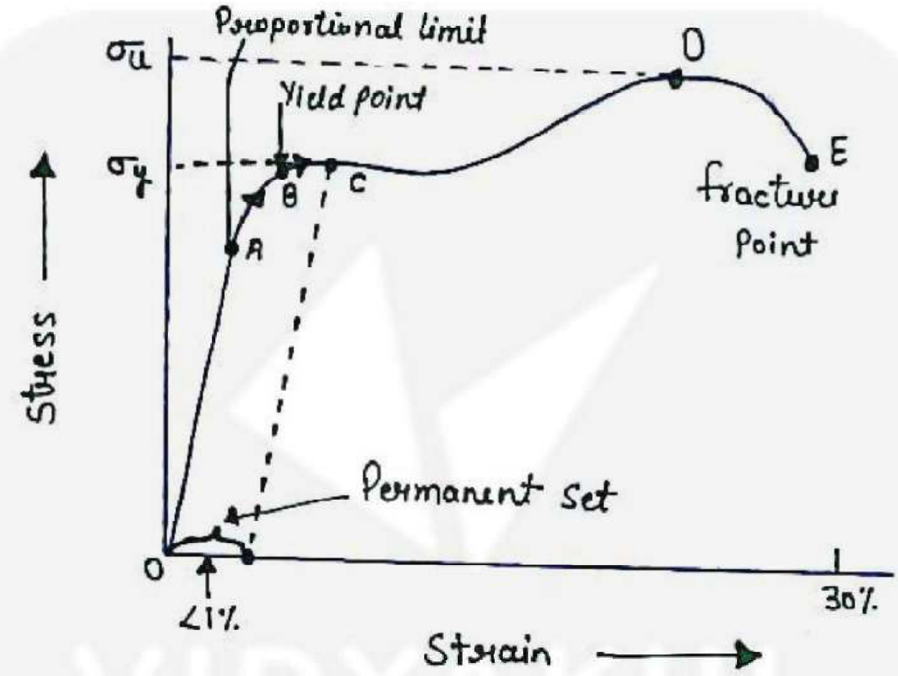
Hooke's Law states that, within elastic limits, the ratio of stress to the corresponding strain produced is a constant. This constant is called the modulus of elasticity.

$$\boxed{\text{Modulus of elasticity} = \frac{\text{Stress}}{\text{Strain}}}$$

Since strain is a pure number, the units of this constant are the same as those of stress i.e., Nm^{-2}.

Stress Strain Curve

Stress Strain Curve are useful to understand the tensile strength of a given material.



The stress - Strain curves can vary with the material.

We can see that in the region between O and A, the curve is linear. Hence, Hooke's Law obeys in this region. In the region from A to B, the stress and Strain are not proportional. However, if we remove the load, the body returns to its original dimension.

The point B in the curve is the yield point or the elastic limit and the corresponding stress is the yield strength (σ_y) of the material. Once the load is increased further, the stress starting exceeding the yield strength. This means that the strain incre--ases rapidly even for a small change in the stress.

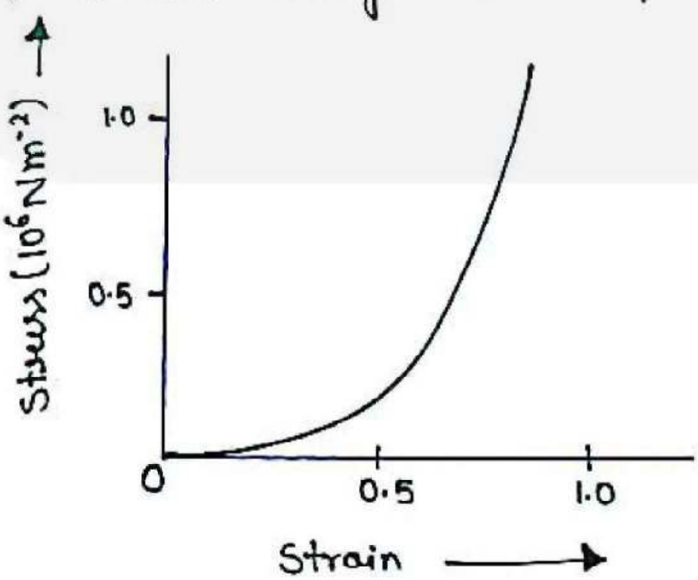
This is shown in the region from B to D in the curve. If the load is removed at, say a point C between B and D, the body does not regain its original dimension. Even when the stress is zero, the strain is not zero and the deformation is called plastic deformation.

Further, the point D is the ultimate tensile strength (σ_u) of the material. Hence if any additional strain is produced beyond this point, a fracture can occur (point E). If,

⇒ The ultimate strength and fracture points are close to each other (point D and E), then the material is brittle.

⇒ The ultimate strength and fracture points are far apart (points D and E), then the material is ductile.

The stress - strain behaviour varies from material to material. Rubber, for example, can be stretched upto several times its original length and it still returns to its original shape.



from the curve, elastic region is very large, the material does not obey Hooke's Law. Also, there is no well-defined plastic region. Materials like rubber, tissue or the aorta, etc. which can be stretched to cause large strains are called elastomers.

Elastic Moduli

In the stress-strain curve, the region within the elastic limit (region OA) is of importance of structural and manufacturing sectors. Since it describes the maximum stress a particular material can take before being permanently deformed.

Elastic Moduli can be three types -

1) Young's modulus
2) Shear modulus
3) Bulk modulus

Young's Modulus

For a solid, in the form of a wire or a thin rod, Young's modulus of elasticity within elastic limit is defined as the ratio of longitudinal stress to longitudinal strain.

Young's Modulus

$$\boxed{Y = \frac{F/A}{\Delta l/l}}$$

$$= \frac{Fl}{A \cdot \Delta l} = \frac{mgl}{\pi r^2 \cdot \Delta l}$$

It has the unit of longitudinal stress and dimensions of $[ML^{-1}T^{-2}]$. Its unit is Pascal or N/m^2.

Shear Modulus or Modulus of Rigidity

Shear Modulus (G) is the ratio of Shearing Stress to the corresponding shearing strain. Another name for shear stress is the Modulus of Rigidity.

$$\boxed{G = \frac{\text{Shearing Stress} (\sigma_s)}{\text{Shearing Strain}}}$$

$$G = \frac{F/A}{\Delta x / L}$$

$$= \frac{F \times L}{A \times \Delta x}$$

We know that, Shearing Strain $= \theta$

$$G = \frac{F/A}{\theta}$$

$$\boxed{= \frac{F}{A \times \theta}}$$

the SI unit of Shear Modulus is N/m^2 or Pa.

Bulk Modulus

Within elastic limit the bulk modulus is defined as the ratio of longitudinal stress and volumetric strain.

Bulk modulus

$$\boxed{B = \frac{F/A}{\Delta V/V}}$$

$$= -\frac{P}{\Delta V/V}$$

(-)ve indicates that the volume variation and pressure variation always negate each other.

Reciprocal of bulk modulus is commonly referred to as the "Compressibility". It is defined as the fractional change in volume per unit change in pressure.

Poisson's Ratio

The ratio of Change in diameter (ΔD) to the original diameter (D) is called lateral Strain. The ratio of Change in length (Δl) to the original length (l) is called longitudinal Strain. The ratio of lateral Strain to the longitudinal strain is called Poisson's Ratio.

$$\boxed{\sigma = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}}$$

$$\boxed{= -\frac{\Delta D / D}{\Delta l / l}}$$

for most of the Substances, the value of σ lies between 0.2 to 0.4. when a body is perfectly incompressible, the value of σ is maximum and equal to 0.5.

Elastic fatigue

It is the property of an elastic body by virtue of which its behaviour becomes less elastic under the action of repeated alternating deforming forces.

Relations between Elastic Moduli

for isotropic materials (i.e., materials having the same properties in all directions), Only two of the three elastic constants are independent.

for example, Young's modulus can be expressed in terms of the bulk and shear moduli.

$$\frac{3}{Y} = \frac{1}{\eta} + \frac{1}{3B}$$

Also,

$$\boxed{Y = 3B(1-\sigma) = 2\eta(1+\sigma)}$$

Breaking Stress

The ultimate tensile strength of a material is the stress required to break a wire or a rod by pulling on it. The breaking stress of the material is the maximum stress which a material can withstand. Beyond this point breakage occurs.

When a wire of original length L is stretched by a length l by the application of force F at the end, then

$$\boxed{\text{Work done to stretch wire} = \frac{1}{2} \times \text{Stretching force} \times \text{extension}}$$

$$= \frac{1}{2} \times \frac{YAl^2}{L}$$

⇒ Work done per unit Volume of wire is given as :

$$\boxed{W = \frac{1}{2} \text{Stress} \times \text{Strain}}$$

According to the formula

$$Y = \frac{F \cdot L}{A l}$$

Where F is the force needed to stretch the wire of length L and area of Cross-Section A. l is the increases in the length of the wire.

$$\boxed{F = \frac{Y \cdot A \cdot l}{L}}$$

The work done by this force in stretching the wire is sorted in the wire as potential energy.

$$dW = F \times dl$$

$$= \frac{Y \cdot A \cdot l}{L} \cdot dl$$

Integrating both Sides,

$$W = \frac{YA}{L} \int_0^l l \cdot dl$$

$$W = \frac{YA}{L} \left[\frac{1}{2} l^2 \right]_0^l$$

$$= \frac{YA}{L} \left[\frac{1}{2} l^2 \right]$$

$$= \frac{1}{2} \left(\frac{YAl}{L} \right) \cdot l$$

$$\boxed{= \frac{1}{2} \cdot F \cdot l}$$

which equal to the elastic potential energy U.

$$U = \frac{1}{2} F.l$$

$$\boxed{= \frac{1}{2} \times \text{Force} \times \text{extension}}$$

Now the potential energy per unit volume is

$$\frac{1}{2} \frac{F.l}{V} = \frac{1}{2}\left(\frac{YAl}{L}\right) \cdot \frac{l}{V}$$

$$\frac{1}{2} \times \frac{Fl}{AL} = \frac{1}{2}\left(\frac{YAl}{L}\right) \cdot \frac{l}{AL}$$

$$[\because V = AL]$$

$$= \frac{1}{2}\left(\frac{Yl}{L}\right) \cdot \frac{l}{L}$$

$$= \frac{1}{2}\left(\frac{F}{A}\right) \cdot \frac{l}{L}$$

$$\boxed{= \frac{1}{2} \times \text{Stress} \times \text{Strain}}$$

Hence, the elastic potential energy of a wire is equal to half the product of its stress ...