Chapter - 5

Lows of Motion

Dynamics

Is the branch of physics in which we study the motion of a body by taking into consideration the cause i.e., force which produces the motion.

Force

Force is an external cause in the form of push or pull, which produces or tries to produce motion in a body at rest, or stops / tries to stop a moving body or changes / tries to change the direction of motion of the body.

The inherent property, with which a body resists any change in its state of motion is called ine-

rtie. Heavier the body, the inertia is more and lighter the body, lesser the inertia.

Law of inertia states that a body has the inability to change its state of rest or uniform motion (i.e., a motion with constant velocity) or direction of motion by itself.

Newtons first Low of Motion

A body will remain at rest or continue to move with uniform velocity unless an external force is applied to it.

First low of motion is also referred to as the 'Law of inertia'. It defines, force and inertial frame of reference. -

There is always a need of 'frame of reference' to describe and und-

erstand the motion of particle. The simplest 'frame of reference' used are known as the inertial frames.

A frame of referent, e is known as an inertial frame it, within it, all accelerations of any particle are caused by the action of 'real forces' on that particle.

When we talk about accelerations produced by 'fictitious' or 'pseudo' forces, the frame of reference is a non-inertial one.

Newton's Second Laws of Motion

When an external force is applied to a body of constant mass the force produces an acceleration, which is directly proportion. which is directly proportional to the force and

inversely proportional to the mass of the body.

Newton's Third Law of Motion

"To every action there is equal and opposite reaction force". When a body A exerts a force on another body B, B exerts an equal and opposite force on A.

Linear Momentum

The linear momentum of a body is defined as the product of the mass of the body and its velocity.

$$\boxed{\text{Linear momentum} = \text{mass} \times \text{velocity}}$$

$$\boxed{\vec{p} = m\vec{v}}$$

where m is the mass of the body, $\vec{v}$ is the velocity of the body and $\vec{p}$ linear momentum. Momentum is a vector quantity having the same direction as the velocity ($\vec{v}$). Its SI unit is kg ms^{-1}.

Impulse

Forces acting for short duration are called impulsive forces. Impulse is defined as the product of force and the small time interval for which it acts. It is given by

$$I = \int F \, dt$$

Impulse of a force is a vector quantity and its SI unit is 1 Nm.

If force of an impulse is changing

with time, then the impulse is measured by finding the area bound by force-time graph for that force.

Impulse of a force for a given time is equal to the total change in momentum of the body during the given time. Thus, we have

$$I = \int_{t_1}^{t_2} \vec{F}\, dt = \vec{P_2} - \vec{P_1}$$

(Impulse-momentum theorem)

Conservation of Momentum

The total momentum of an isolated system of particles is conserved. In other words, when no external force is applied to the system, its total momentum remains constant.

If $F_{ext} = 0$, $\sum_{i=1}^{n} P_i = $ constant or $P = $ constant

Recoiling of a gun. flight of rockets and jet planes are some simple applications of the law of conservation of linear momentum.

When a bullet of mass m_b is fired w. a velocity $\vec{v}_b$, the gun of mass m_g will acquire a velocity $\vec{v}_g$ which is given by

$$\vec{v}_g = -\frac{m_b\,\vec{v}_b}{m_g}$$

The negative sign shows that direction of $\vec{v}_g$ is opposite to the direction of $\vec{v}_b$ i.e., the gun recoils.

The rocket sent up in space will acquire a velocity (v) which is given by

$$v = v_0 + u \log_e \frac{m_0}{m}$$

(Instantaneous velocity of the rocket)

Usually, initial velocity of the rocket at $t = 0$ is zero i.e., $v_0 = 0$

Thus,

$$v = u \, \log_e \frac{m_0}{m}$$

Exhaust speed (u) of the gases.

Log of the ratio of initial mass (m_0) of the rocket to its mass (m) at that instant of time.

Speed of the rocket at any instant depends upon these two factors discussed above.

where, m_0 = initial mass of the rocket and fuel.

$\vec{v_0}$ = initial velocity of rocket

m = mass of the rocket at any instant t during its flight

$\vec{v}$ = velocity acquired by the rocket

at that instant w.r.t. ground.

The instantaneous acceleation of the rocket is given by

$$a = -\frac{u}{m}\frac{dm}{dt}$$

where $\frac{dm}{dt}$ is the rate at which the fuel is consumed.

Equilibrium of a Particle

A group of forces which are acting at one point are called concurrent forces."

Concurrent forces are said to be in equilibrium if there is no change in the position of rest or the state of uniform motion of the body on which these concurrent forces are acting.

For concurrent forces to be in equilibrium, their resultant force must be zero. In case of three concurrent forces acting in a plane, the body will be in equilibrium if these forces may be completely represented by three sides of a triangle taken in order. If number of concurrent forces is more than three, then these forces must be represented by sides of a closed polygon in order of equilibrium.
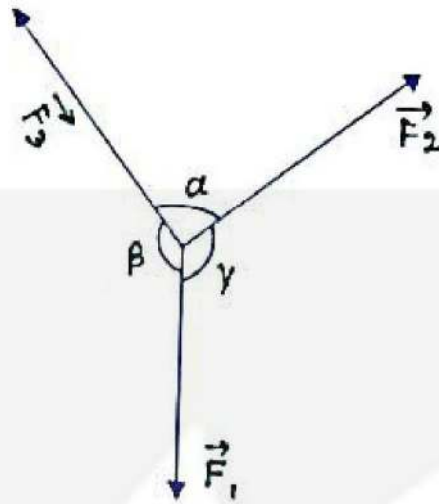
Mathematically

$$\sum \vec{F} = 0 \text{ i.e. } \vec{F_1} + \vec{F_2} + \vec{F_3} + \ldots\ldots = 0 .$$

If three concurrent forces $\vec{F_1}$, $\vec{F_2}$ and $\vec{F_3}$ act at a point and are in equilibrium, then

$$\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}$$

where a, β and γ are angles indicated.

Common forces in Mechanics

Commonly Used Forces

(i) Weight of a body.

It is the force with which earth attracts a body towards its cen-tre. If M is mass of body and g is acceleration due to gravity, weight of the body is Mg in ver-tically downward direction.
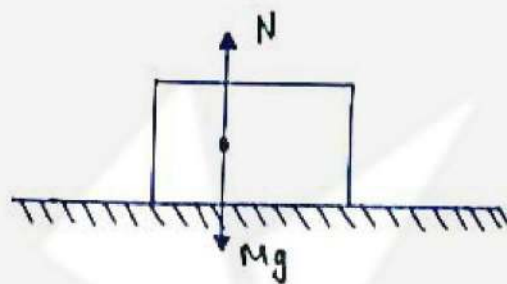
ii) Normal Force

If two bodies are in contact a contact force arises, if the surface is smooth the direction of force is normal to the plane of contact. We call this force as Normal force.
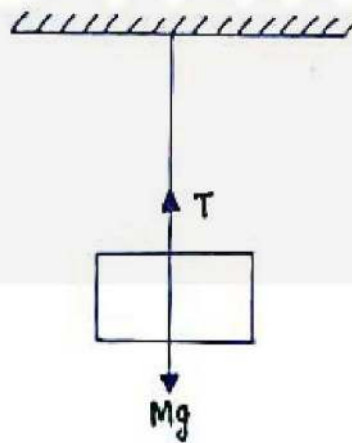

Example

Let us consider a book resting on the table. It is acted upon by its weight in vertically downward direction and is at rest. It means there is another force acting on the block in opposite direction, which balances its weight. This force is provided by the table and we call it as normal force.

(iii) Tension in string

Suppose a block is hanging form a string. Weight of the block is acting vertically downward but it is not moving. hence its weight is balanced by a force due to string. This force is called 'Tension in string'.

Tension is a force in a stretched string. Its direction is taken along the string and away from the body under consideration.
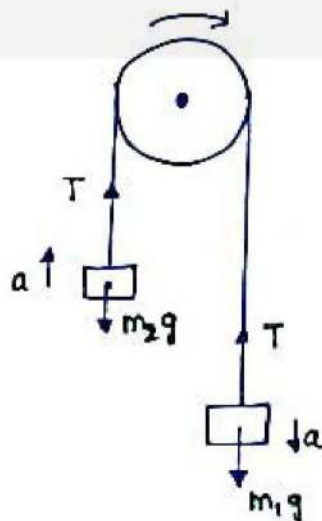

Simple Pulley

Consider two bodies of masses m_1 and

m_2 tied at the ends of an inextensible string, which passes over a light and friction less pulley. Let $m_1 > m_2$. The heavier body will move upwards. Let a be the common acceleration of the system of two bodies. which is given by.

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2}$$

Tension in the string is given by

$$T = \frac{2\,m_1\,m_2 \times g}{m_1 + m_2}$$

Apparent Weight and Actual Weight

'Apparent weight' of a body is equal to its 'actual weight' if the body is either in a state of rest or in a state of uniform motion.

Apparent weight of a body for vertically upward accelerated motion is given as

Apparent weight = Actual weight + Ma = M (g + a)

Apparent weight of a body for vertically downward accelerated motion is given as

Apparent weight = Actual weight Ma = M (g - a)

Friction

The opposition to any relative motion between two surfaces in contact is

referred to as friction It arises because of the two surfaces in contact.

Static and Dynamic (Kinetic) Friction

The frictional forces between two surfaces in contact (i) before and (ii) after a relative motion between them has started. are referred to as static and dynamic friction respectively. Static friction is always a little more than dynamic friction.

The magnitude of Kinetic frictional force is also proportional to normal force.

$$f_K = \mu_k N$$

Limiting Frictional Force

This frictional force acts when body

is about to move. This is the maximum frictional force that can exist at the contact surface. We calculate its value using laws of friction.

Laws of Friction

(i) The magnitude of limiting frictional force is proportional to the normal force at the contact surface.

$$\boxed{f_{lim} \propto N \Rightarrow f_{lim} = \mu_s N}$$

Here μ_s is a constant, the value of which depends on nature of surfaces in contact and is called as 'coefficient of static friction'. Typical values of μ ranges from 0.05 to 1.5.

(ii) The magnitude of limiting frictional force is independent of area of

contact between the surfaces.

Coefficient of Friction

The coefficient of friction (μ) between two surfaces is the ration of their limiting frictional force to the normal force between them, i.e.,

$$\mu = \frac{\text{limiting frictional force}}{\text{normal reaction}} = \frac{F}{R}$$

When there is no relative motion $\mu = \mu_s$, the coefficient of static friction. When there is a relative motion $\mu = \mu_k$, the coefficient of kinetic friction.

Angle of Friction

It is the angle which the resultant

of the force of limiting friction F and the normal reaction R makes with the direction of the normal reaction. If θ is the angle of friction, we have
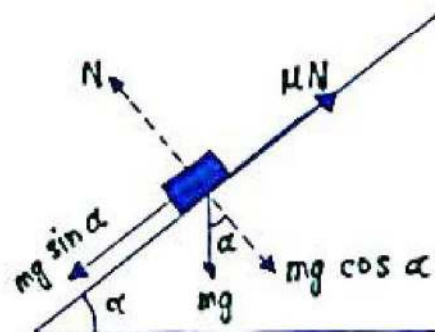
$$\boxed{\tan \theta = \frac{F}{R} = \mu}$$

$$\boxed{\theta = \tan^{-1} \mu}$$

Circular Motion

Angle of Repose

Angle of repose (α) is the angle of an inclined plane with the horizontal at which a body placed over it just begins to slide down without any acceleration. Angle of repose is given by $\alpha = \tan^{-1}(\mu)$

Motion on a Rough Inclined Plane

Suppose a motion up the plane takes place under the action of pull P acting parallel to the plane.

Centripetal Force

Centripetal force is the force required to move a body uniformly in a circle. This force acts along the radius and towards the centre of the circle. It is given by

$$N = mg \cos \alpha$$

Frictional force acting down the plane,

$$\boxed{F = \mu N = \mu \, mg \, \cos \alpha}$$

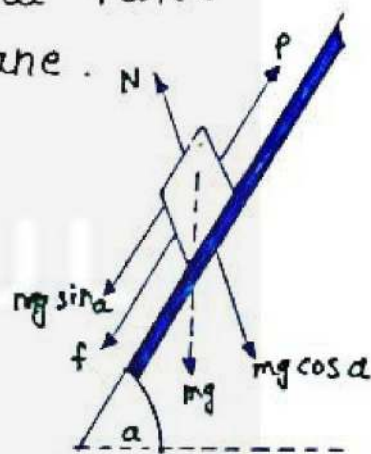Applying Newton's second law for motion up the plane.

$$P - (mg \sin \alpha + f) = ma$$

$$P - mg \sin \alpha - \mu \, mg \cos \alpha = ma$$

If $P = 0$ the block may slide downwards with an acceleration a. The frictional force would then act up the plane.

$$\boxed{mg \sin \alpha - F = ma}$$

or, $mg \sin \alpha - \mu \, mg \cos \alpha = ma$

Friction can be reduced by
(i) making the surfaces smooth, (ii) lubrication,
(iii) replacing sliding friction by rolling friction, (iv) using ball bearings, (v) streamlining the shape of the bodies.

Where, v is the linear velocity, r is the radius of circular path and ω is

the angular velocity of the body.

Centrifugal Force

Centrifugal force is a force that arises when a body is moving actually along a circular path, by virtue of tendency of the body to regain its natural straight line path.

The magnitude of centrifugal force is same as that of centripetal force.

$$F = \frac{mv^2}{r} = mr\omega^2$$

. The maximum speed with a vehicle can negotiate a curve is $v_{max} = \sqrt{\mu r g}$, where μ is the coefficient of friction and r is the radius of the path.

. In order to help a vehicle owner to drive safely along a curved path, the outer edge of the road is slightly raised above the inner edge. The angle of banking

θ of the road from horizontal is given by

$$\tan \theta = \frac{v^2}{rg} \quad \text{or} \quad v \leq \sqrt{rg \tan \theta}$$

- In a banked curve (θ) with friction, the safe velocity is given as

$$v = \left[\frac{rg(\mu + \tan \theta)}{1 - \mu \tan \theta} \right]^{1/2}$$
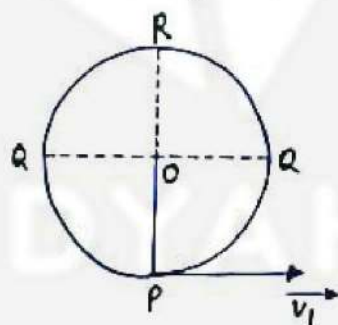
Motion in a Vertical Circle

The motion of a particle in a horizontal circle is different from the motion in vertical circle. In horizontal circle, the motion is not effected by the acceleration due to gravity (g) whereas in the motion of vertical circle, the motion is not effected by the acceleration due to gravity (g) whereas

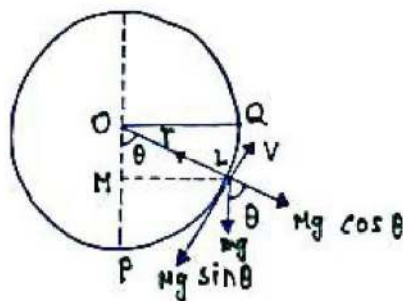in the motion of vertical circle. the value of 'g' plays an important role. the motion in this case does not remain uniform When the particle move up from its lowest position P. its speed continuously decreases till it reaches the highest point of its circular path. This is due to the work done against the force of gravity. When the particle moves down the circle, its speed would keep on increasing.

Let us consider a particle moving in a circular vertical path of radius V and centre o tide. with a string. L be the instantaneous position of the particle such that

$$\angle POL = \theta°$$

Here the following force act on the particle of mass 'm'

(i) Its weight = mg (verticaly downwards)

(ii) The tension in the string T along LO.

The instantaneous velocity of the particle at L is $\overline{V}$ along the direction of the tangent to the circle.

Centripetal force on the particle $= \dfrac{mV^2}{r}$

$\therefore \qquad \dfrac{mV^2}{r} = T - mg \cos\theta \quad [\text{From Fig.(ii)}]$

So $\qquad \boxed{T = \dfrac{mV^2}{r} + mg \cos\theta}$

We can take the horizontal direction at the lowest point 'p' as the position of zero gravitational potential energy. Now as per the principle of conservation of energy.

The total energy at P = Total energy at L

$$\frac{1}{2}mV_1^2 + 0 = \frac{1}{2}mV^2 + mgh$$

$\Rightarrow$ $\boxed{V_1^2 = V^2 + 2mgh}$ (ii)

From right angled Δ OML,

$$OM = OL \cos\theta$$
$$= r \cos\theta$$
$$MP = h = OP - OM$$
$$= r - r\cos\theta$$
$$= r(1-\cos) \quad (iii)$$

Substituting the value of 'h' in eq.(ii) we get,

$$\boxed{V_1^2 = V^2 + 2gr(1-\cos\theta)}$$

Now substituting the value of V^2 from (iv) to (i), we get

$$T = \frac{m}{r}\left[V_1^2 - 2gr(1-\cos\theta)\right] + mg\cos\theta$$
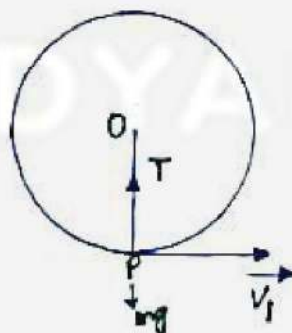
$$T = \frac{mV_1^2}{r} - 2mg(1-\cos\theta) + mg\cos\theta$$

$$= \frac{mV^2}{r} - 2mg + 2mg\cos\theta + mg\cos\theta$$

$$= \frac{mV_1^2}{r} - 2mg + 3mg\cos\theta \qquad (v)$$

From this relation, we can calculate the tension in the string at the lowest point P, mid-way point and at the highest position of the moving particle.

Case (i)

At the lowest point P, $\theta = 0°$

$$T_P = \frac{mV_1^2}{r} - 2mg + 3mg\cos 0°$$

$$= \frac{mV_1^2}{r} - 2mg + 3mg$$

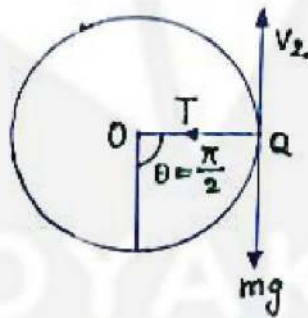$$= \frac{mV_1^2}{r} + mg \qquad (vi)$$

Hence, the tension in the string at the lowest point P is,

$$\boxed{T = \frac{mV_1^2}{r} + mg}$$

Case (ii)

At the point $'Q'$ where the string in horizontal position. Where $\theta = \frac{\pi}{2}$

$$T_Q = \frac{mV_1^2}{r} - 2mg + 3mg \cos\frac{\pi}{2}$$

$$= \frac{mV_1^2}{r} - 2mg + 0$$

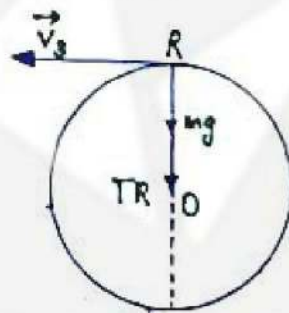He $\qquad T_Q = \frac{mV_1^2}{r} - 2mg \qquad\qquad (iii)$

The change in the tension the string at two position P and Q

$$= (T_P - T_Q)$$

$$= \left(\frac{mV_1^2}{r} + mg \right) - \left(\frac{mV_1^2}{r} - 2mg \right)$$

$$= 3 \, mg$$

Case (iii)

At point R, the highest position of the moving particle where $\theta = \pi$.

$$T_R = \frac{mV_1^2}{r} - 2mg + 3mg \cos \pi$$

$$= \frac{mV_1^2}{r} - 2mg - 3mg$$

Here

$$\boxed{T_R = \frac{mV_1^2}{r} - 5mg} \qquad (viii)$$

The change in the string as the particle moves from P to R.

$$= (T_P - T_R)$$

$$= \left(\frac{mV_1^2}{r} + mg \right) - \left(\frac{mV_1^2}{r} - 5\,mg \right) = 6\,mg$$

Thus we can see that the tension in the string is maximum at the lowest point P and minimum at tha highest point R of the circular path.

From Eq. (viii) it is clear that T_R may be (a) pasitive (b) negative (c) zero, depending on the value of $\bar{V}_1$. If T_R becomes a negative mumber, the string would get slackened and the particle will fall down without completing the circular path. Therefore, to complete the circular path, the minimum value of T_R must be zero.

$$T_{R(min)} = \frac{mV_1^2 (min)}{r} - 5\,mg$$

$$0 = \frac{mV_1^2 (min)}{r} - 5\,mg$$

$$V_{1(min)} = \sqrt{5gr}$$

When the particle completes its motion along the vertical circle, it is referred to as "Looping the Loop" for this the minimum speed at the lowest position must be $\sqrt{5gr}$