Chapter - 4

Motion in a plane

Motion

In a plane is called as motion in two dimensions. e.g. projectile motion, Circular motion etc. For the analysis of such motion our reference will be made of an origin and two co-ordinate axes X and Y.

Scalar and Vector Quantities

Scalar Quantities - The Physical quantities which are completely specified by their magnitude or size alone are called scalar quantities.

Examples - Length, mass, density, speed, work etc.

Vector Quantities - Vector quantities are those physical quantities which are characterised by both magnitude and direction.

Examples: Velocity, Displacement, Acceleration, force, momentum, torque etc.

Characteristics of Vectors

Unit Vector

A Unit Vector is a vector of unit magnitude and points in a particular direction. It is used to specify the direction Only. Unit Vector is represented by putting a Cap (^) Over the quantity.

The unit vector in the direction of $\vec{A}$ is denoted by $\hat{A}$ and defined by

$$\boxed{\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A}} \quad \text{or}$$

$$\boxed{\vec{A} = A \cdot \hat{A}}$$

Equal Vectors

Vectors $\vec{A}$ and $\vec{B}$ are said to be equal if $|\vec{A}| = |\vec{B}|$ as well as their directions are same.

Zero Vectors

A Vector with zero magnitude and an arbitrary direction is Called a zero Vector. It is represented by $\vec{0}$ and also known as "null Vector."

Negative of a vector

The Vector whose magnitude is same as that of $\vec{a}$ but the direction is opposite to that of $\vec{a}$ is called the negative of $\vec{a}$ and is written as $-\vec{a}$.

Parallel Vectors

$\vec{A}$ and $\vec{B}$ are said to be parallel vectors if they have same direction and may or may not have equal magnitude ($\vec{A} \parallel \vec{B}$). If the directions are opposite, then $\vec{A}$ is anti-parallel to $\vec{B}$.

Displacement Vector

The displacement vector is a vector which gives the position of a point with reference to a point Other than the Origin of the co-ordinate system.

$$\boxed{\vec{r}_{12} = \vec{r}_2 - \vec{r}_1}$$

Parallelogram Law of Vector Addition

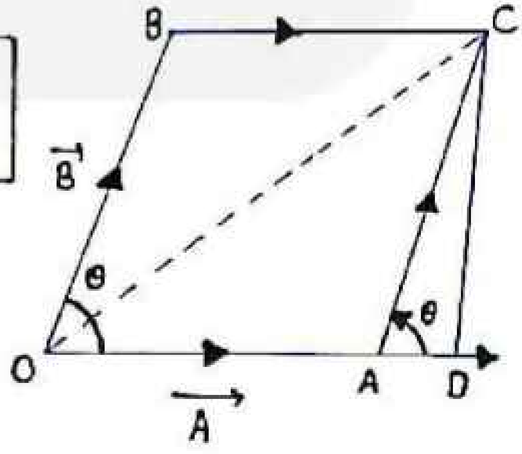
If two vectors, acting simultaneously at a point, can be represented both in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then the resultant is represented completely both in magnitude and direction by the diagonal of the parallelogram passing through that point.

If $\vec{A}$ and $\vec{B}$ be two adjacent sides of a parallelogram, inclined at angle θ, then the magnitude of resultant vector is

$$\boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

Direction of resultant $\vec{R}$ – Let α be the angle made by resultant $\vec{R}$ with vector $\vec{A}$. then

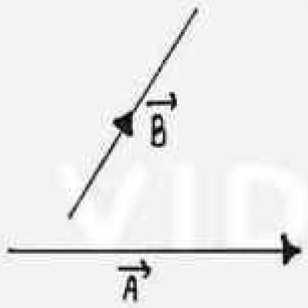
$$\boxed{\alpha = \tan^{-1}\frac{B\sin\theta}{A + B\cos\theta}}$$

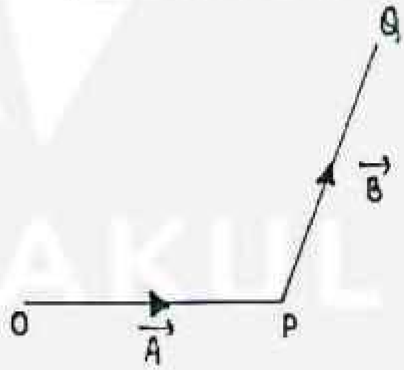


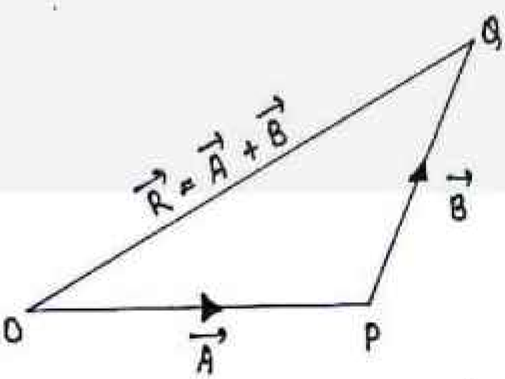
Triangle Law of Vector Addition

If two vectors are represented both in magnitude and direction by the two sides of a triangle taken in the same order, then the resultant of these vectors is represented both in magnitude and direction by the third side of the triangle taken in the opposite order.

$\boxed{\overrightarrow{OP} = \vec{A}}$ and $\boxed{\overrightarrow{PQ} = \vec{B}}$







The sum of the resultant of $\vec{A}$ and $\vec{B}$ is represented by the vector $\overrightarrow{OQ}$ (Joining tail of $\overrightarrow{OP}$ to the head of $\overrightarrow{PQ}$.)

Hence,

$$\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$$
$$OQ = \vec{A} + \vec{B}$$

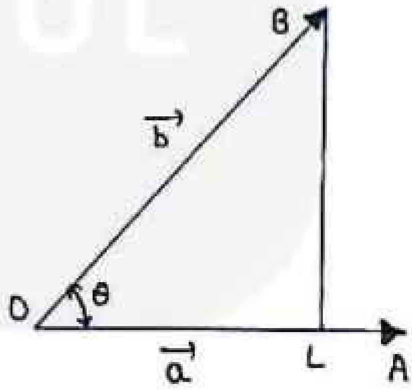
Multiplication of Vectors by real Numbers

⇒(i) Scalar product (Dot product)

Scalar product of two vectors is defined as the product of the magnitude of two vectors with cosine of smaller angle between them.

It is always a scalar, so it is called as scalar product

$$\boxed{\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta}$$



Geometrically,

$\vec{a} \cdot \vec{b}$ = (Mod of $\vec{a}$)(Projection of b on $\vec{a}$)

(ii) Vector Product (Cross product)

The cross or vector product of two vectors $\vec{A}$ and $\vec{B}$ is defined as,

$$\boxed{\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta \, \hat{n}}$$

where θ – Angle between $\vec{A}$ and $\vec{B}$ taken in anti-clockwise direction.

$\hat{n}$ – unit vector in the direction perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

Geometrically, $\vec{a} \times \vec{b}$ is a vector whose modulus is the area of the parallelogram formed by the two vectors as the adjacent sides and direction is perpendicular to both $\vec{a}$ and $\vec{b}$.

Properties of Scalar product

(i) It Obeys commutative law

$$\boxed{\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}}$$

(ii) It Obeys distributive Law

$$\boxed{\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}}$$

(iii) Scalar (Dot) Product of two mutually perpendicular vectors is zero i.e.,

$$\boxed{(\vec{A}\cdot\vec{B}) = AB\cos 90^\circ = 0}$$

(iv) Scalar (Dot) Product will be maximum when $\theta = 0^\circ$ i.e., vectors are parallel to each other.

$$\boxed{(\vec{A}\cdot\vec{B})_{max} = |A||B|}$$

(v) If $\vec{a}$ and $\vec{b}$ are unit vectors then

$$|\vec{a}||\vec{b}| = 1 \text{ and } \vec{a}\cdot\vec{b} = 1\cdot 1\cos\theta = \cos\theta$$

(vi) Dot Product of unit vectors $\hat{i}, \hat{j}, \hat{k}$

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

(vii) Square of a vector

$$\vec{a}\cdot\vec{a} = |a||a|\cos 0 = a^2$$

(viii) If the two vectors $\vec{A}$ and $\vec{B}$, in terms of their rectangular components, are

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \text{ and } \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k},$$

then, $\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$$\boxed{\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z}$$

Properties of Vector Addition

(i) It obeys Commutative Law

If $\vec{a}$ and $\vec{b}$ are any two vectors, then

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

(ii) It obeys associative Law

If $\vec{a}, \vec{b}$ and $\vec{c}$ are any three vectors, then

$$\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$$

(iii) It obeys distributive property

If $\vec{a}$ and $\vec{b}$ are two vectors and λ is a real number then

$$\lambda(\vec{a} + \vec{b}) = \lambda\vec{a} + \lambda\vec{b}$$

Resolution of Vectors

It is a process of Splitting vector into two or more vectors in different directions which together produce the same effect as is produced by the Single vector alone.

The vectors into which the given Single vector is splitted are called Component of vectors. In fact, the resolution of a vector is just opposite to Composition of vectors.

(i) If a vector $\vec{A}$ makes an angle θ with x-axis then magnitude of its rectangular Components in x-y plane are given by

$A_x = A\cos\theta$ and $A_y = A\sin\theta$ where,

$$\boxed{A = \sqrt{A_x^2 + A_y^2}}$$

(ii) If a vector $\vec{A}$ lie in free space and Subtends an angle α with x-axis, angle β with y-axis and angle γ with z-axis then the magnitudes of its rectangular Components along the three axes are given as $A_x = A\cos\alpha$, $A_y = A\cos\beta$ and

$A_z = A \cos \gamma$. where.

$$\boxed{A = \sqrt{A_x^2 + B_y^2 + C_z^2}}$$

(iii) A vector $\vec{A}$ may be expressed in terms of its rectangular components as:

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along x, y and z axis respectively.

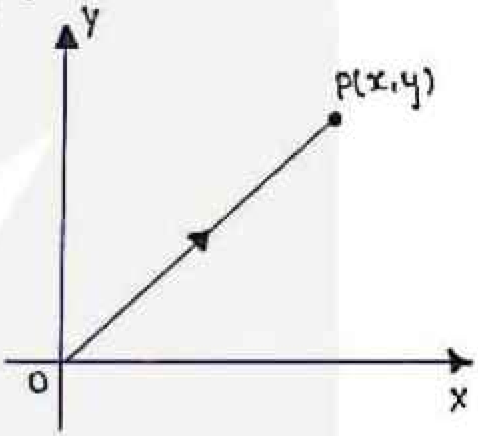
If the components of a given vectors are perpendicular to each other, then they are called rectangular components.

Motion in a plane

Position Vector

Position vector is a vector to represent any position of a body. The straight line joining the origin and the point represents the position vector. It is repres-ented by both magnitude and direction.

It is represented by $\vec{r} = \vec{OP} = x\hat{i} + y\hat{i}$ where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axis respectively.



If position vector $\vec{r}$ is in three dimensions, then it is given by $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ where, $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors alongs x, y and z co-ordinates respectively.

$$\boxed{|\vec{r}| = \sqrt{x^2 + y^2 + z^2}}$$

Angular Displacement

Angular displacement of the Object moving around a circular path is defined as the angle traced out by the radius vector at the centre of the circular path in a given

Time.

$$\boxed{\theta\,(\text{angle}) = \frac{\text{arc}}{\text{radius}}}$$

θ = the magnitude of angular displacement.
It is expressed in radians (rad).

Angular Velocity

Angular Velocity of an Object in circular motion is defined as the time rate of Change of its angular displacement.
It is denoted by ω and is measured in radians per second (rad s^{-1}).

$$\boxed{\omega = \frac{\text{angular displacement}}{\text{Time}} = \frac{\theta}{t} = \frac{d\theta}{dt}}$$

Angular Accelaration

Angular accelaration of an object in circular motion is defined as the time rate of Change of its angular velocity.

It is denoted by 'α' and measured in rad s^{-2}.

$$\boxed{\alpha = \frac{\text{angular Velocity change}}{\text{time taken}} = \frac{d\omega}{dt}}$$

for uniform angular acceleration α, the equations of motion can be modified as,

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

Projectile Motion

The projectile is a general name given to an object that is given an initial inclined Velocity and which Subsequently follows a path determined by the gravitational force acting on it and by the frictional resistance of the air. The path followed by a projectile is called its trajectory.

Equation of Projectile Motion

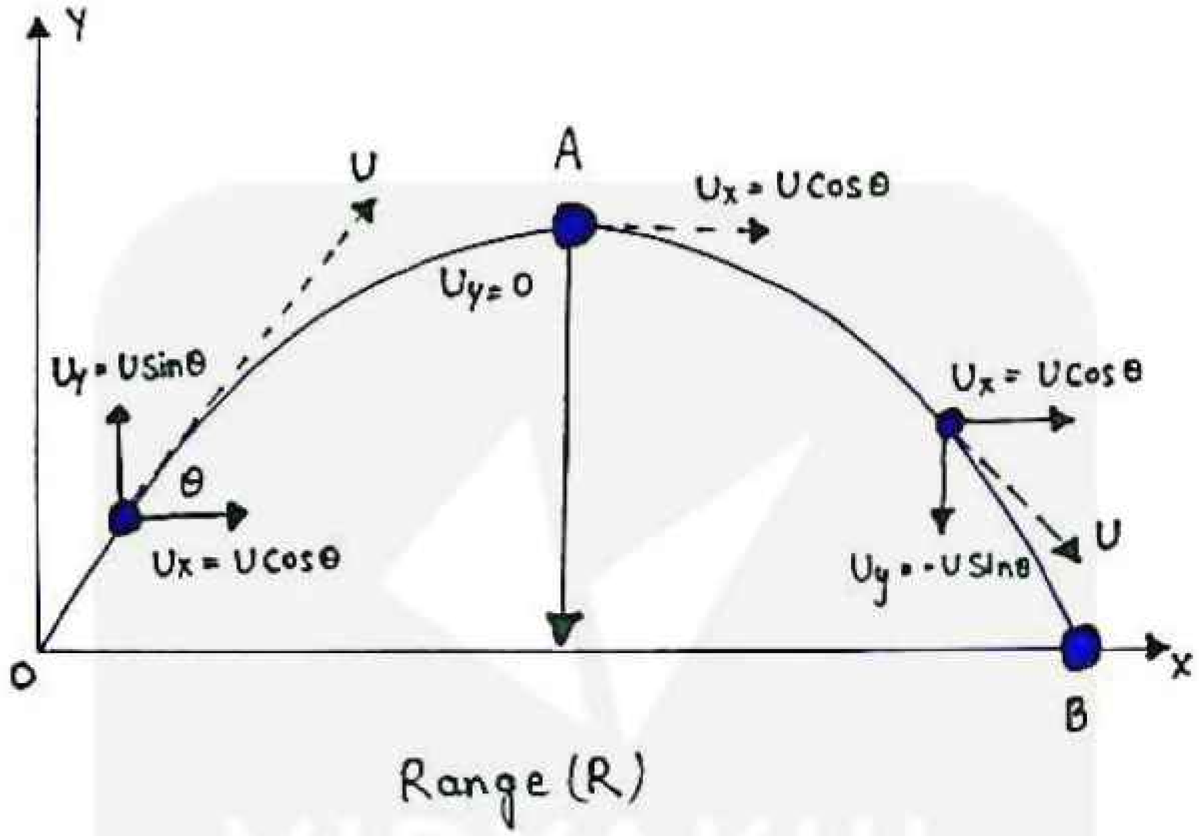
The general case of projectile Motion corresponds to that of an object that has been given an initial Velocity u at some angle θ above (or below) the horizontal. The horizontal and vertical displacements x and y are given by

$$u_x = u \cos\theta$$

$$u_y = u \sin\theta$$

Then the equation of trajectory of a projectile is given as

$$\boxed{y = (\tan\theta)x - \frac{g}{2(u\cos\theta)^2}x^2}$$



Y
U
A
Ux = UCosθ
Uy= 0
Uy = USinθ
Ux = UCos θ
θ
U
Ux = UCosθ
Uy = -USinθ
O
X
B
Range (R)

The above equation is in the form of $y = ax + bx^2$ where a and b are constants. This is a equation of a parabola. Thus the trajectory of a projectile is parabolic.

Time of Flight - The time taken by a projectile to return to its initial elevation after projection is known as its time to flight (T). It is given by

$$\boxed{T = \frac{2u \sin\theta}{g}}$$

Horizontal Range - The maximum horizontal distance between the point of projection and the point on the horizontal plane where the projectile hits is called horizontal range.

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

The range of the projectile will be maximum if Sin 2θ is maximum (i.e. 1)

$$\sin 2\theta = 1$$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$

Thus the projectile has maximum Range if it is projected at an angle of 45° with the horizontal.

$$\boxed{R_{max} = \frac{u^2}{g}}$$

Maximum Height - The maximum vertical distance travelled by the projectile during its journey is called the maximum hight attained by the projectile.

$$\boxed{H = \frac{u^2 \sin^2\theta}{2g}}$$

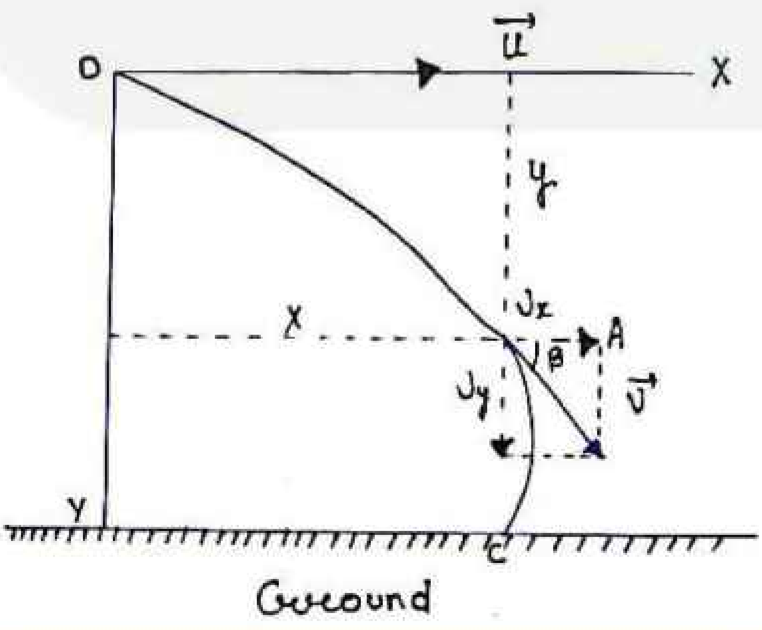
Projectile Given Horizontal Projection

(i) Equation of Path $y = kx^2$, which is a parabola

(ii) Time of flight $T = \sqrt{\frac{2h}{g}}$

(iii) Horizontal Range
$$R = u\sqrt{\frac{2h}{g}}$$

(vi) Velocity at any time t is $V = \sqrt{u^2 + g^2t^2}$ and angle made by resultant velocity with horizontal
$$\beta = \tan^{-1}\left(\frac{gt}{u}\right)$$



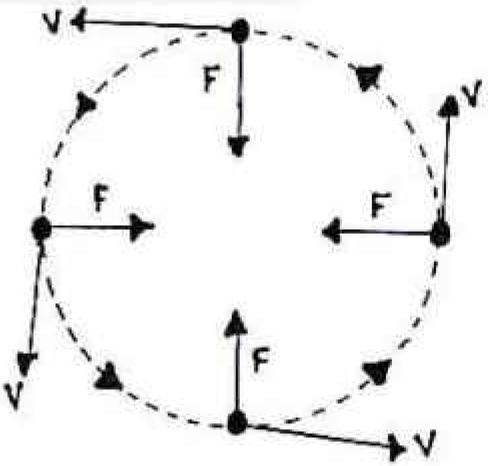
(v) Velocity of projectile when it hits the ground

$$\boxed{v = \sqrt{u_1^2 + 2gh}}$$

Uniform Circular Motion

When a body moves in circular path with a Constant Speed, then the motion of the body is known as uniform Circular Motion.

The time taken by the object to complete one revolution on its circular path is called time period. For circular Motion, the number of revolutions completed per unit time is known as the frequency (ν). Unit of frequency is 1 Hertz (1 Hz). It is fount that



$$\nu \cdot T = 1$$

$$\boxed{\nu = \frac{1}{T}}$$