# Chapter -3

## Motion in a Straight Line

### Introduction

Motion is one of the significant topics in physics. Everything in the universe moves. It might only be a small amount of movement and very-very slow, but movement does happen. Even if you appear to be standing still, the Earth is moving around the sun, and the sun is moving around our galaxy.

" An object is said to be in motion if its position changes with time ".

The concept of motion is a re'live one and a body that may be in motion relative to one reference system, may be at rest relative to another.

There are two branches in physics that examine the motion of an object.

(i) Kinematics : It describes the motion of objects, without looking at the cause of the motion.

(ii) Dynamics : It relates the motion of objects to the forces which cause them.

## Instantaneous Speed and Instantaneous Velocity

Instantaneous Speed. The speed of an object at an instant of time is called instantaneous speed.

"Instantaneous speed is the limit of the average speed as the time interval becomes infinitesimally small".

$$\bar{v} = \lim \frac{\Delta \bar{x}}{\Delta t} = \frac{d\bar{x}}{dt}$$

Instantaneous velocity

The instantaneous velocity of a particle is the velocity at any instant of time or at any point of its path.

"Instantaneous velocity or simply velocity is defined as the limit of the average velocity as the time interval $\Delta t$ becomes infinitesimally small."

$$v = \lim_{\Delta t \to 0} \frac{\Delta \bar{x}}{\Delta t} = \frac{dx}{dt}$$

## Acceleration

The rate at which velocity changes is called acceleration.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

$$a = \frac{v - u}{t}$$

where $v$ and $u$ are final and initial

velocity respectively. It is a vector quantity with S.I. unit of $m/s^2$ and has dimensions of $[LT^{-2}]$.

If acceleration is -ve (negative), then it is called retardation or deceleration.

## Uniform Acceleration

If an object undergoes equal changes in velocity in equal time intervals it called uniform acceleration.

## Averege and Instantaneous Acceleration

Average Accelerating. It is the change in the velocity divided by the time-interval during which the change occurs.

$$\vec{a} = \frac{\Delta v}{\Delta t}$$

Instantaneous Acceleration. It is defined as the limit of the average acceleration as the time-interval $\Delta t$ goes to zero.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{dv}{dt}$$

## Kinematical Graphs

The 'displacement – time' and the 'velocity – time' graphs of a particle are often used to provide us with a visual representation of the motion of a particle. The 'shape' of the graphs depends on the initial 'co – ordinates' and the 'nature' of the acceleration of the particle (Fig.)

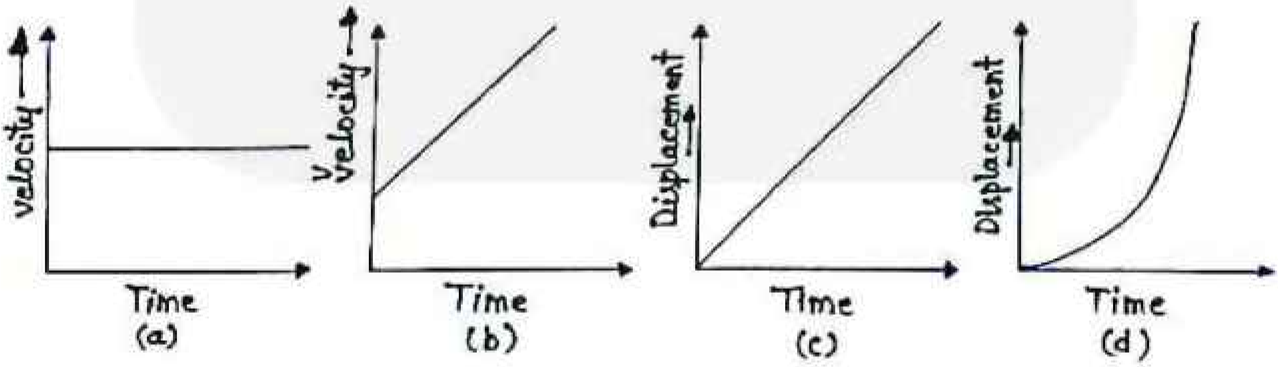



Fig. Curves (a) and (c) represent motion with a constatt speed v. Curves (b) and (d) represent motion with a uniform acceleration a starting with an initial speed u.

The following general results are always valid

(i) The slope of the displacement-time graph at any instant gives the speed of the particle at that instant.

(ii) The slope of the velocity-time graph at any instant gives the magnitude of the acceleration of the particle at the instant.

(iii) The area enclosed by the velocity-time graph, the time-axis and the two co-ordinates at, time instants $t_1$ to $t_2$ gives the distance moved by the particle in the time-interval from $t_1$ to $t_2$.

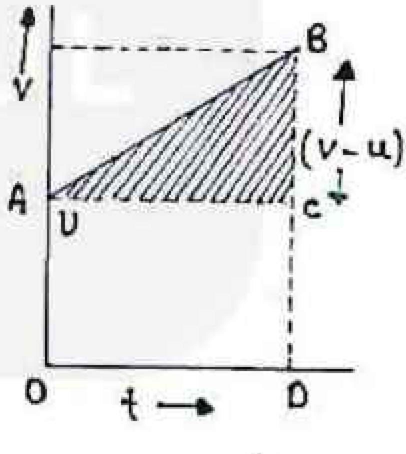
## Equations of Motion for Uniformly Accelerated Motion

For uniformly accelerated motion, some simple equations can be derived that relate displacement (X), time taken (f). initial velocity (u), final velocity (v) and acceleration (a). Following equation gives a relation between final and initial velocities v and u of an object moving with uniform acceleration a: $v = u + at$

This relation can be graphically represented by following figure:

The area under this curve is:
Area between instants 0 and t

= Area of triangle ABC + Area of rectangle OACD




$$= \frac{1}{2}(v-u)t + ut$$

The area under v − t curve represents the displacement. Therefore, the displacement x of the object is:

$$x = \frac{1}{2}(v-u)t + ut$$

But $\quad v - u = at$

So, $\quad x = \frac{1}{2} at^2 + ut$

or, 

$$\boxed{x = ut + \frac{1}{2} at^2}$$

The equation for displacement can also be given as follows :

$$x = \frac{v + u}{2} t = vt$$

Earlier we have derived:

$$\boxed{v = u + at}$$

or, $\quad \frac{v + u}{a} = t$

Substituting the value of t in equation for displacement we get,

$$x = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{2}\right)$$

or, $$x = \frac{v^2 - u^2}{2a}$$

or, $$\boxed{v^2 = u^2 + 2ax}$$

So, we have derived following kinematic equation.

$$v = u + at$$

$$x = ut + \frac{1}{2}at^2$$

$$v = u^2 + 2ax$$

If acceleration is uniform or constant, then equations of motion are:

| | |
|---|---|
| $v = u + at$ | .....(i) |
| $s = ut + \frac{1}{2}at^2$ | ....(ii) |
| $v^2 = u^2 + 2as$ | ....(iii) |

where u is initial velocity, v is final velocity, a is acceleration and s is the distance covered in time interval t.

For uniformly accelerated motion along a straight line, displacement in a particular instant of time ($n^{th}$ second of the motion) is given by

$$S_{nth} = u + \frac{1}{2} a (2n - 1)$$

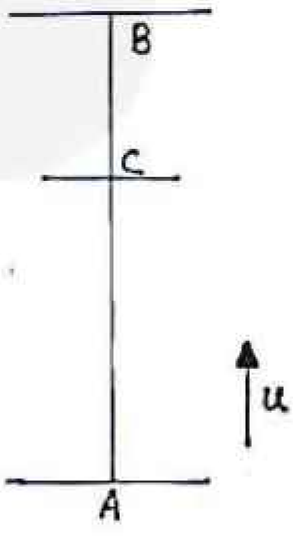
Suppose a body is projected vertically upward from a point A with velocity u.

If we take upward direction as positive

(i) At time t, its velocity $v = u - gt$

(ii) At time t, its displacement from A is given by

$$h = ut - \frac{1}{2} g t^2$$




(iii) Its velocity when it has a displacement 'h' is given by

$$v^2 = u^2 - 2gh$$

(iv) When it reaches the maximum height from A, its velocity $v = 0$. This happen when $t = \frac{v}{g}$. The body is instantaneously at the highest points.

(v) The maximum height reached

$$H = \frac{u^2}{2g}$$

(vi) Total time to go up and return to the point of projection $= \frac{2u}{g}$.

(vii) At any point C between A and B, where $AC = S$, the velocity $v$ is given by

$$v = \pm\sqrt{u^2 - 2gs}$$

The velocity of body while crossing C upwards $= +\sqrt{u^2 - 2gs}$ and while crossing C downwards is $-\sqrt{u^2 - 2gs}$.

In some problems it is convenient to take the downward direction as positive, in such case all the measurments in downward direction are considered as positive i.e.. acceleration will be + g. But sometimes we may need to take upward as positive and if such case acceleration will be - g.

## Relative Velocity

Relative velocity of an object A with respect to another object B is the time rate at which the object A changes its position with respect to the object B.

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$, where $\vec{V}_A$ and $\vec{V}_B$ are the velocities of object A and B. ($\vec{V}_A - \vec{V}_B$) indicates the addition of negative of velocity of B to the velocity of A.

→ The relative velocity of two objects

moving in the same direction is the differnce of the speeds of the objects.

→ The relative velocity of two objects moving in opposite direction is the sum of the speeds of the objects.

## Some physical quantities, symbols, dimensions and their units.

| S.No. | Physical quantity | Symbol | Dimensions | Units |
|---|---|---|---|---|
| (i) | Path lengh | | $[L]$ | m |
| (ii) | Displacement | $\Delta x$ | $[L]$ | m |
| (iii) | Velocity<br>(a) Average<br>(b) Instantaneous | <br>$\bar{v}$<br>$v$ | $[LT^{-1}]$ | $ms^{-1}$ |
| (iv) | Speed<br>(a) Average<br>(b) Instantaneous | | $[LT^{-1}]$ | $ms^{-1}$ |
| (v) | Acceleration<br>(a) Average<br>(b) Instantaneous | <br>$\bar{a}$<br>$a$ | $[LT^{-2}]$ | $ms^{-2}$ |