Chapter - 2

Unit and Measurement

Measurement

The process of measurement is basically a comparison process. To measure a physical quantity. we have to find out how many times a standard amount of that physical quantity is present in the quantity being measured. The number thus obtained is known as the magnitude and the standard chosen is called the unit of the physical quantity.

Unit

The unit of a physical quantity is an arbitrarily chosen standard which is widely accepted by the society and in terms of which other quantities of similar nature may be measured.

Standard

The actual physical embodiment of the unit of a physical quantity is known as a standard of that physical quantity.

. To express any measurement made we need the numerical value (n) and the unit (μ). Measurement of physical quantity = Numerical value x Unit

For example: Length of a rod = 8 m where 8 is numerical value and m (metre) is unit of length.

Fundamental Physical Quantity / Units

It is an elementary physical quantity, which does not require any other physical quantity to express it. It means it cannot be resolved further in terms of any other physical quantity. It is also know as basic physical quantity.

The units of fundamental physical quantities are called fundamental units. For example, in M.K.S. system, Mass, Length and Time expressed in kilogram metre and second respectively are fundamental units.

Derived Physical Quantity / Units

All those physical quantities, which can be derived from the combination of two or more fundamental quantities or can be expressed in terms of basic physical quantities, are called derived physical quantities.

The units of all other physical quantities, which can be obtained from fundamental units, are called derived units. For example, units of velocity, density and force are m/s, kg/m3, Kg m/s2 respectively and they are examples of derived units.

Systems of Units

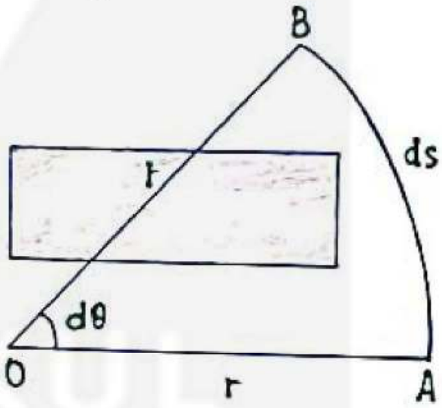
Earlier three different units systems were in different countries. These were CGS, FPS and MKS systems. Now-a-days internationally SI system of units is followed. In SI unit system, seven quantities are taken as the base quantities.

(i) CGS System. Centimetre, Gram and Second are used to express length, mass and time respectively.
(ii) FPS System. Foot, pound and second are used to express length, mass and time respectively.
(iii) MKS System. Length is expressed in metre, mass is expressed in kilogram and time is expressed in second. Metre, Kilogram and second are used to express length, mass and time respectively.
(iv) SI Units. Length, mass, time, electric current, thermodynamic temperature, Amount of substance and luminous intensity are expressed in metre, Kilogram, second, kelvin, mole and

candela respectively.

Supplementary Units

Besides the above mentioned seven units, there are two supplementary base units. thers are (i) radian (rad) for angle, and (ii) steradian (sr) for solid angle.



(i) **Radian (rad)**. It is the unit of plane angle. One radian is an angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.

$$\boxed{d\theta = \left(\frac{ds}{r}\right) \text{ radian}}$$

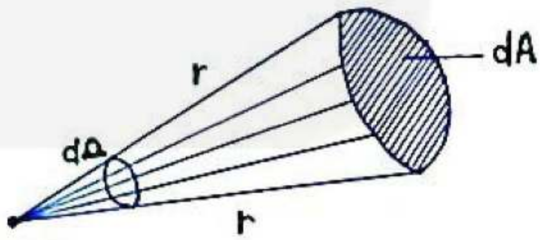
(ii) **Steradian (sr)**. It is the unit of solid angle. One steradian is the solid angle subtended at the centre of a sphere by its surface whose area is equal to the square of the radius of the sphere. Solid angle in steradian,

$$d\Omega = \frac{\text{area cut out from the surface of sphere}}{(\text{radius})^2}$$

$$\boxed{d\Omega = \left(\frac{dA}{r^2}\right) \text{ steradian}}$$

Advantages of SI Unit System

SI Unit System has following advantages over the other Besides the above mentioned seven units, there are two supplementary base units. These are systems of Units :



(i) It is internationally accepted,
(ii) It is a rational unit system,
(iii) It is a coherent unit system,
(iv) It is a metric system,
(v) It is closely related to CGS and MKS systems of units.

(vi) Uses decimal system, hence is more user friendly.

Other Important Units of Length

For measuring large distances e.g., distances of planets and stars etc., some bigger units of length such as 'astronomical unit', 'light year', parsec' etc. are used.

. The average separation between the Earth and the sun is called one astronomical unit.

$$\boxed{1 \text{ AU} = 1.496 \times 10^{11} \text{ m.}}$$

. The distance at which an arc of length of one astronomical unit subtends an angle of one second at a point is called parsec.

$$\boxed{1 \text{ parsec} = 3.08 \times 10^{16} \text{ m}}$$

. The distance travelled by light in vacuum in one year is called light year.

$$\boxed{1 \text{ light year} = 9.46 \times 10^{11} \text{ m}.}$$

- Size of a tiny nucleus = 1 fermi = 1f $= 10^{-15}$ m
- Size of a tiny atom = 1 angstrom = 1 Å $= 10^{-10}$ m

Accuracy, Precision of instruments and Errors in measurement

Absolute Error, Relative Error and Percentage Error

- If $a_1, a_2, a_3, \ldots\ldots, a_n$ be the measured values of a quantity in several measurements, then their mean is considered to be the true value of that quantity i.e:

$$\text{true value } a_0 = a_{mean} = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}$$

- The magnitude of the difference between the true value of the quantity and the individual measurement value is the absolute error of that measurement. Hence, absolute errors in measured values are :

$$\Delta a_1 = a_0 - a_1, \Delta a_2 = a_0 - a_2, \Delta a_3 = a_0 - a_3, \ldots\ldots$$

$$\ldots\ldots, \Delta a_n = a_0 - a_n$$

- The arithmetic mean (i.e., the mean of the magnitudes) of all the absolute errors is known as the mean absolute error.

$$\Delta a_{mean} = \frac{[|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \ldots + |\Delta a_n|]}{n}$$

- The ratio between mean absolute error and the mean value is called relative error.

$$\text{Relative error} = \frac{\text{Mean absolute error}}{\text{Mean value}}$$

$$= \frac{\Delta a_{mean}}{a_{mean}} = \frac{\Delta a_{mean}}{a_0}$$

- Percentage error is the expression of the relative error in percentage.

$$\boxed{\text{Percentage error} = \frac{\Delta a_{mean}}{a_{mean}} \times 100\%}$$

Combination of Errors

- If a quantity Z be expressed as the sum or difference of two quantities A and B (i.e., if $Z = A + B$ or $Z = A - B$), then maximum value of error $\Delta Z = \Delta A + \Delta B$.

Hence, when two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.

- If a quantity Z be expressed as product or a quotient of quantities A and B, then the maximum fractional error in Z is given by.

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Hence, when two quantities are multiplied or divided, the relative error in the result is the sum of the relative errors in the multipliers.

- If $Z = A^m B^n C^l$ etc., then maximum fractional error in Z is given by

$$\boxed{\frac{\Delta Z}{Z} = m\frac{\Delta A}{A} + n\frac{\Delta B}{B} + l\frac{\Delta C}{C}}$$

Significant Figures

The significant figures are a measure of accuracy of a particular measurement of a physical quantity.

Significant figures in a measurement are those digits in a physical quantity that are known reliably plus the first digit which is uncertain.

The Rules for Determining the Number of Significant Figures

(i) All non-zero digits are significant.

(ii) All zeroes between non-zero digits are significant.

(iii) All zeroes to the right of the last non-zero digit are not significant in numbers without decimal point.

(iv) All zeroes to the right of a decimal point and to the left of a non-zero digit are not significant.

(v) All zeroes to the right of a decimal point and to the right of a non-zero digit are significant.

(vi) In addition and subtraction, we should retain the least decimal place among the value operated, in the result.

(vii) In multiplication and division, we should express the result with the least number of significant figures as associated with the least precise number in operation

(viii) If scientific notation is not used :

(a) For a number greater then 1, without any decimal, the traiting zeroes are not significant.

(b) For a number with a decimal, the trailing zeros are significant.

Dimensions

The dimensions of a physical quantity are the powers to which the fundamental units of mass, length and time must be raised to represent the given physical quantity.

Dimensional Formule

The dimensional formula of a physical quantity is an expression telling us how and which of the fundamenal quantities enter into the unit of that quantity.

It is customary to express the

fundamental quantities by a capital letter, e.g., length (L), mass (M), time (T), electric current (I), temperature (K) and luminous intensity (C). We write appropriate powers of these capital letters within square brackets to get the dimensional formule of any given physical quantity.

Applications of Dimensions

The concept of dimensions and dimensional formulae are put to the following uses:

(i) Checking the results obtained
(ii) Conversion from one system of units to another
(iii) Deriving relationships between physical quantities
(iv) Scaling and studying of models.

The underlying principle for these uses is the principle of homogeneity of dimensions. According to this principle, the 'net' dimensions of the various physical quantities on both sides of a permissible physical relation must be the same; also only dimensionally similar quantities can be added to or subtracted from each other.

- If a given physical quantity has a dimensional formula $M^a L^b T^c$, then

$$\boxed{n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c}$$

Where M_1, L_1, T_1 and M_2, L_2, T_2 are the units of mass, length and time in two systems and n_1 and n_2, the numerical values of the physical quantity in these unit systems.

Limitations of Dimensional Analysis

The method of dimensions has the following limitations:

(i) by this method the value of dimensionless constant cannot be calculated.

(ii) by this method the equation containing trigonometric, exponential and logarithmic terms cannot be analyzed.

(iii) If a physical quantity in mechanics depends on more than three factors, then relation among them cannot be established because we can have only three equations by equalizing the powers of M, L and T.

(iv) It doesnt tell whether the quantity is vector or scalar.

Dimensional Formulae of Some Physical Quantities

Physical Quantity	Dimensional Formula	Physical Quantity	Dimensional Formula
Area	L^3	Capacitance	$M^{-1}L^{-2}T^2Q^2$
Volume	L^3	Electric current	I or QT^{-1}
Density	ML^{-3}	Electric potential	$ML^2T^{-2}Q^{-1}$

Velocity	LT^{-1}		or $ML^2T^{-3}I^{-1}$
Acceleration	LT^{-2}	Electric field	$ML^{-2}Q^{-1}$
Momentum	MLT^{-1}		or $MLT^{-3}I^{-1}$
Force	MLT^{-2}	Inductance	ML^2Q^{-2}
Energy, work	ML^2T^{-2}		or $ML^2T^{-2}I^{-2}$
Power	ML^2T^{-3}	Resistance	$ML^2T^{-1}Q^{-2}$
Frequency	T^{-1}		or $ML^2T^{-3}I^{-2}$
Pressure	$ML^{-1}T^{-2}$	Magnetic flux	$ML^2T^{-1}Q^{-1}$
Torque, couple	ML^2T^{-2}		or $ML^2T^{-2}I^{-1}$
Moment of inertia	ML^2	Magnetic field vector H	$L^{-1}T^{-1}Q$
Temperature	K		or $L^{-1}I$
Heat energy	ML^2T^{-2}	Magnetic feild intensity, B	$MT^{-1}Q^{-1}$
Entropy	$ML^2T^{-2}K^{-1}$		or $MT^{-2}I^{-1}$
Specific heat capacity	$L^2T^{-2}K^{-1}$	Permeability	MLQ^{-2}

Specific latent heat	L^2T^{-2}		or M
Thermal conductivity	$MLT^{-3}K^{-1}$	Permittivity	$M^{-1}L^{-3}T^2Q^2$
Electric charge	Q or IT		or $M^{-1}L^{-3}T^4I^2$