Chapter - 15

Waves

Most of us experience the phenomenon of Wave propogation when we drop a stone in a pond of still water. These waves move outwards in expanding circles until they reach the shore. It seems as if the water is moving outward in expanding circles until they reach the shore. It seems as if the water is moving outward from the point of disturbance

Thus, we can say that energy is transferred but there is no transfer of medium. Such a pattern in which there is no actual transfer of matter as a whole but energy is transmitted from one part of a medium to another part is called Wave.

Mechanical Waves

Mechanical waves can be produced or propagated only in a material medium. These waves are governed by Newton's laws of motion.

For example, waves on water surface, waves on strings, sound waves etc.

Electromagnetic Waves

These are the waves which require no material medium for their production and propogation. i.e. they can pass through vacuum and any other material medium.

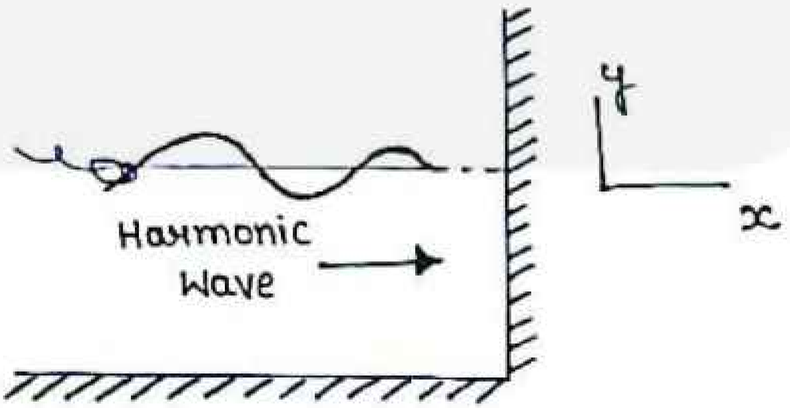
Example- waves are visible light ; ultra-violet light ; radiowaves, microwaves etc.

Transverse and Longitudinal waves

Transverse Waves

Transverse waves are the waves in which the constituents of the medium oscillate perpendicular to the direction of wave propagation.

If we give an upward jerk to one end of a long rope that has its opposite end fixed, a single wave pulse is formed and travels along the rope with a fixed speed.



Longitudinal Waves

Longitudinal waves are the waves in which the constituents of the medium oscillate along the direction of wave propagation.

a longitudinal wave is the one that move parallel to the direction of waves of particles in motion.

For instance in the same rope kept horizontally, if one introduce a pulse on the left and the right end, the energy flows from both ends trapping the movements in a parallel motion. These are longitudinal waves.

Displacement Relation in a Progressive Wave -

A traveling wave is also referred as a progressive wave. It may be transverse or longitudinal. We consider a transverse wave so that if it travels along positive x-axis, the constituent particles of the medium vibrate along y-axis, about their mean positions. In general, sinusoidal shape can be described by sine function or cosine function.

At time t, the displacement y of the element located at position x is given by

$$\boxed{y_{(x,t)} = a \sin(kx - \omega t + \phi)}$$

Where, $y_{(x,t)}$ = Displacement of the medium particles from their mean positions

a = Amplitude of a wave

ω = Angular frequency of the wave

k = Angular wave number

ϕ = Initial phase angle

Amplitude and Phase

Amplitude

The maximum displacement of the Constituents of the medium from their equilibrium position is called Amplitude of Wave.

Phase

Phase of a wave is the argument of the Sine function. Phase determines the displacement of the wave at any position and at any instant.

Crest

It is the point of maximum positive dis--placement on a wave. The peak of a Sinu--soidal wave represents a Crest.

Trough

It is the point of maximum negative displa-cement on a wave.

Wave Velocity :-

Wave velocity is the time rate of propagation of wave motion in the given medium. It is different from particle velocity. Wave velocity depends upon the nature of medium.

Wave Velocity = frequency X wavelength

$$\boxed{(v) = (\nu) \times (\lambda)}$$

Wavelength

The wavelength 'λ' for a progressive wave is basically the distance measured between two succeeding points of the same phase at a particular time. Considering a stationary wave, this is twice the distance measured between two successive nodes or antinodes. The propagation constant is defined as 'k'. The SI unit is calculated in radian per meter or rad s^{-1}.

$$\boxed{k = \frac{2\pi}{\lambda}}$$

Period, Angular Frequency and Frequency

The time period 'T' of wave oscillation is the duration taken by any component of the medium takes to travel over one

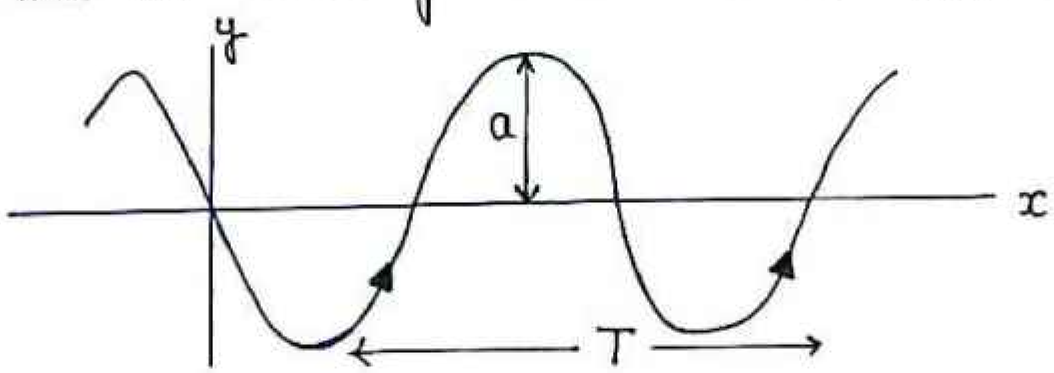
Complete Oscillation. This is related to 'ω' or Angular frequency through the following relation :

$$\boxed{\omega = \frac{2\pi}{T}}$$

In this, wave frequency 'ν' is mentioned as $\frac{1}{T}$ and is also related to angular frequency as :

$$\boxed{\nu = \frac{\omega}{2\pi}}$$

It can also be defined as the number of oscillations / unit time prepared in a String element while the wave passes through it. This is usually calculated in Hetz.



Frequency of the Stretched String –

In general, if the string vibrates in P loops, the frequency of the string under that mode is given by

$$\boxed{\nu = \frac{P}{2L}\sqrt{\frac{T}{\mu}}}$$

Based on this relation three Law of transverse vibrations of stretched strings arise. They are law of length, Law of tension and law of mass.

Law of Length

The frequency ν is inversely proportional to, the length L of the stretched string.

$$\nu \propto \frac{1}{L}$$

$$\boxed{\nu L = a \text{ (Constant)}}$$

Law of Tension

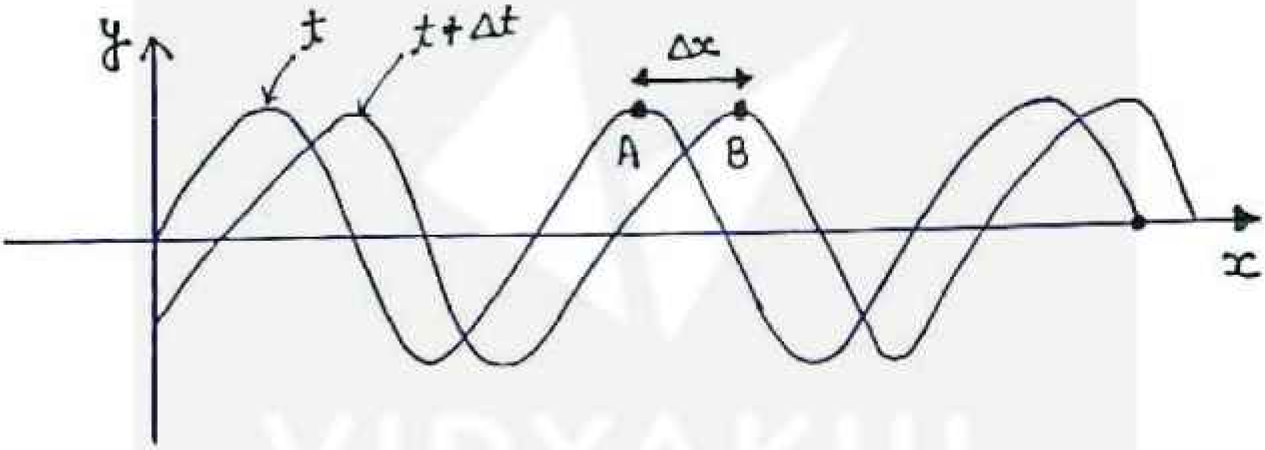
The fundamental frequency is directly proportional to the square root of the tension in the string.

$$\nu = \sqrt{T}$$

$$\boxed{\frac{\nu}{\sqrt{T}} = a \text{ (Constant)}}$$

The speed of a Travelling Wave

If we focus on a particular point on the wave and observe the motion of this point in time, We can determine the speed of a traveling wave.



Here in the figure we have two snapshots of a single traveling wave taken in a small interval Δt. The wave speed can be calculated using the relation

$$V = \frac{\Delta x}{\Delta t}$$

Let us consider a crest shown by point

A in the figures. As the time changes, the position of the crest changes from A to B. Since the phase remains constant

$$kx - \omega t = k(x + \Delta x) - \omega(t + \Delta t)$$

$$kx - \omega t = kx + k\Delta x - \omega t - \omega \Delta t$$

$$k\Delta x = \omega \Delta t$$

$$\frac{\Delta x}{\Delta t} = \frac{\omega}{k}$$

Taking Δx and Δt very small,

$$v = \frac{dx}{dt} = \frac{\omega}{k}$$

Hence, the speed of the wave is

$$\boxed{v = \frac{\omega}{k}}$$

Also, we have, $\omega = \frac{2\pi}{T}$ & $k = \frac{2\pi}{\lambda}$

$$\boxed{v = \frac{\lambda}{T} \quad \text{or} \quad v = f\lambda}$$

Speed of a transverse wave on stretched string -

The speed of a mechanical wave depends on the inertial and elastic properties of the medium. It is inversely related to the inertial property and directly to the elastic property of medium. In case of a stretched string inertial property is its linear mass density (μ). Further, we cannot send a wave along a string unless the string is under a tension. Therefore, we can associate the tension with the elasticity of the string.

Speed of transverse waves on a stretched string is given by

$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$

Speed of a longitudinal Wave

In longitudinal waves, the constituents of the medium oscillate along the direction of wave propagation. The speed of a longitudinal waves, also depends on inertial property as well as elastic property of the medium. The longitudinal waves, such as sound, travel in the form of compressions and rarefactions. Therefore, bulk modulus of the medium can be associated with the elastic property of the medium. Further inertial property relevent for the propogation of these waves is the mass density (ρ) of the medium.

Speed of longitudinal waves

$$\boxed{v = \sqrt{\frac{B}{\rho}}}$$

In case of a linear medium like a solid rod the relevant modulus of elasticity is Young's modulus (Y).

Therefore, In case of Solids, velocity of longitudinal wave is given by

$$\boxed{v = \sqrt{\frac{Y}{\rho}}}$$

Note - The speed of Sound is greater in Solids than in gases even though they are denser than gases. This happens because they are much more difficult to Compress than gases and so have much higher values of bulk modulus.

Speed of Sound in gases

Newton assumed that when sound waves propogate through a gas, the change in pressure and volume of the gas are iso-thermal. The amount of heat produced during compressions is lost to the surroundings and the amount of heat lost during rarefactions is gainted from the surroundings.

Therefore, the temperature of the gas remains constant.

$$\boxed{v = \sqrt{\frac{P}{\rho}}}$$

This relation is known as Newton's formula.

Laplace Correction

Newton assumed that the pressure variations in a medium during propagation of Sound are isothermal. Laplace pointed out that the pressure variations in the gases when Sound propagates are so fast that the heat does get enough time to flow to surroundings or from surroundings to keep the temperature constant. Therefore, the variations are adiabatic and not isothermal.

The speed of sound in a gas is

$$\boxed{v = \sqrt{\frac{\gamma P}{\rho}}}$$

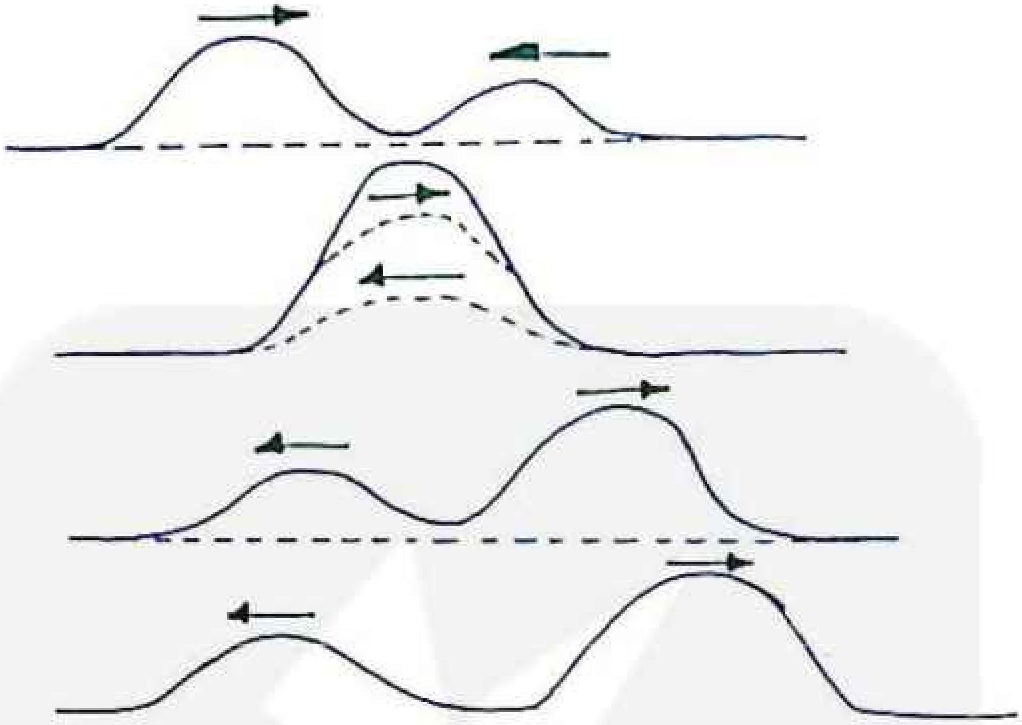
This value of the speed of Sound matches with the speed of Sound measured experimentally.

The principle of Superposition of waves

Whenever two wave pulses travelling in opposite directions cross each other. they retain their individual identities. However at the instant, they overlap each other, the wave pattern is different from the individual pulses. The resultant displacement, at the instant they overlap, is the algebraic sum of the displacement due to each pulse.

Thus we can say that each pulse moves as if others are not present. The net displacement of the Constituents of the medium is an algebraic sum of the two interfering pulses. This is called the Principle of Superposition of waves.

Principle of Superposition



The principle of superposition is expressed by affirming that overlapping waves add algebraically to create a resultant wave. Based on the principle, the overlapping waves (with the frequencies $f_1, f_2, --- f_n$) do not hamper the motion or travel of each other. Therefore, the wave function (y) labeling the disturbance in the medium can be denoted as:

$$y = f_1(x - vt) + f_2(x - vt) + --- + f_n(x - vt)$$

$$\boxed{= \sum_{i=1}^{n} f_i (x - vt)}$$

Hence, the superposition of waves can load to the following three effects:

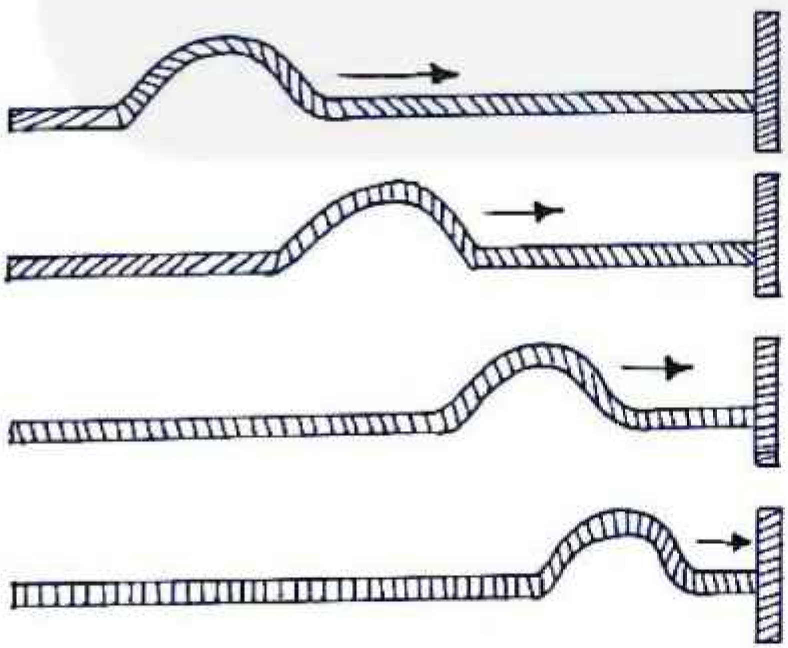
1. Whenever two waves having the same frequency travel with the same speed along the same direction in a specific medium, then they superpose and create an effect known as the interference of waves.

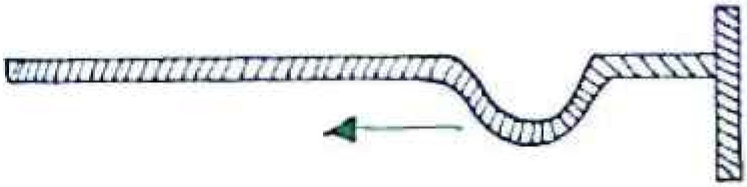
2. In a situation where two waves having similar frequencies move with the same speed along opposite directions in a specific medium, then they superpose to produce stationary waves.

3. Finally, when two waves having slightly varying frequencies travel with the same speed along the same direction in a specific medium, they superpose to produce beats.

Reflection of Waves

When a progressive wave, like a transverse wave travelling along a stretched string arrives at a rigid boundary, the wave gets reflected. The reflected wave suffers a phase change of 180° on reflection. At the rigid boundary, disturbance must have the zero displacement all the time. By the principle of superposition, this could be possible only when the two waves (the incident and the reflected waves) have a phase difference of 180° or π radian.





If we apply Newton's 3rd Law, the arriving wave exerts a force on the rigid boundary. The reaction to this force, exerted by the rigid boundary on the string 'kicks back' on the string and set up a reflected pulse with a phase difference of π radian. Thus, a crest is reflected as a trough.

The phenomenon of echo is an example of reflection of Sound by a rigid boundary.

Standing waves and Normal Modes

When a travelling wave in one direction will get reflected at one end, which in turn will travel and get reflected from the other end. This will go on until there is a steady wave pattern set up on the string. Such wave patterns are

Called Standing waves or Stationary waves. The points, where the Oscillation amplitude is zero are called displacement nodes and the points where the displacement amplitude is maximum are called displacement anti-nodes. Thus, the particles at nodes do not move at all, hence prohibiting any flow of energy through them.

Beats

When two harmonic sound waves of equal Amplitude but slightly different frequencies superpose, the resultant wave look like a single sinusoidal wave with a varying amplitude that goes from maximum to zero and back. The amplitude variation causes the variation of intensity called beats. The frequency with which the amplitude rises and falls is called the beat frequency. and is equal to the difference in frequencies of the two waves. The rise and fall of the intensity of sound is called Waxing and waning.

Let us consider two different sound waves, A and B of varying frequencies but having a similar amplitude propogation in the same medium. When these two sound waves encounter, a fluctuating sound can be heard. Do note that, for a certain time, the crest of A meets the crest of B. Hence, this cause constructive interference.

Therefore, the sound intensity rises for this certain period. However, after a point of time, the crest of B meets the trough of A. In this scenario, destructive inter--ference is experienced; this cause the intensity of Sound to fall for a certain period.

Observations

⇒ The intensity of Sound increases and decreases continuously with time.

⇒ Dissimilar to the original sound wave, the resultant sound wave has an amplitude which isn't constant ; it differs with respect to time.

⇒ Also, whenever the Sound intensity rises to maximum, it is called as waxing of Sound. Conversely, when the sound intensity falls to a minimum, it is termed as the waning of Sound.

⇒ Overall phenomenon of periodic waxing & waning of Sound, where two Sound waves holding almost equal frequencies encounter each Other is called beats.

Beat Frequency can be called as the number of beats generated per second, which is equivalent to the variance in frequencies of two sound waves. This has been converted into an equation for gaining a better understanding.

$$\boxed{f_b = |f_1 - f_2|}$$

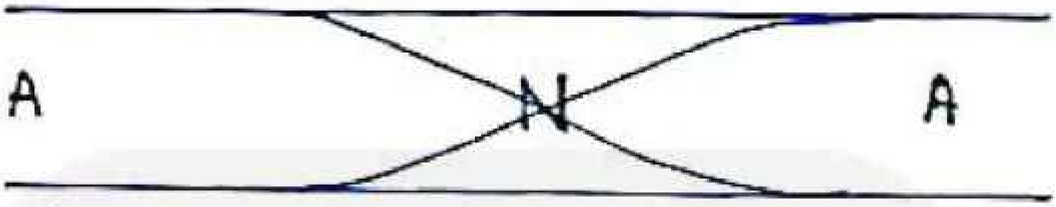
where

f_1 = Sound wave 1 frequency

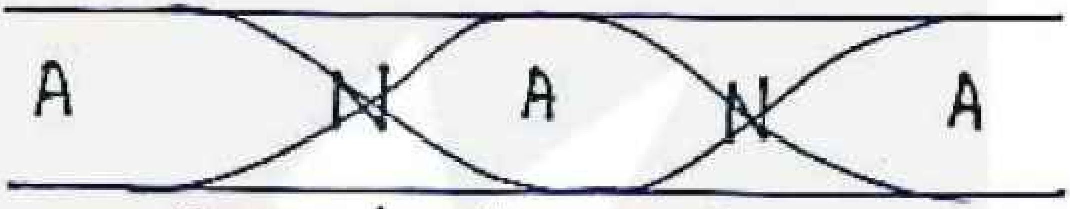
f_2 = Sound wave 2 frequency

$|f_1 - f_2|$ = Difference (positive value)

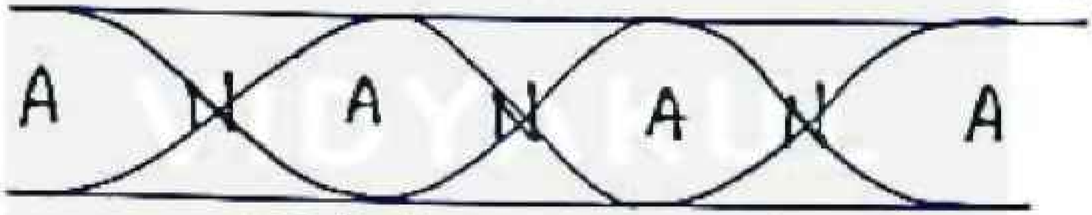
f_b = beats frequency



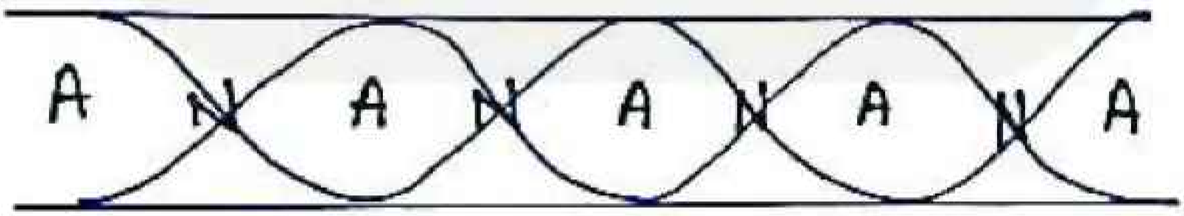
Fundamental or first harmonic



Second Harmonic



Third Harmonic



Fourth Harmonic

Doppler Effect

Whenever there is a relative motion between the source of sound and an observer, the frequency of sound received or heard by the observer is different from the frequency of sound produced by the source. This is called Doppler Effect.

Doppler effect can be observed when

1. The source is moving but the observer is stationary.

2. The observer is moving but the source is stationary.

3. Both the source and observer are moving.

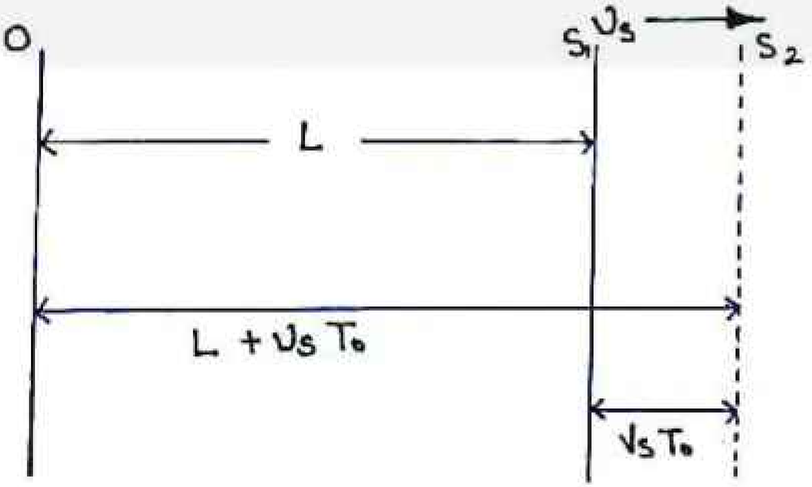
(i) Source Moving : observer Stationary

Let us Consider a Source of Sound produ-
-cing a wave of angular frequency ω, Velocity v and time period T_0, moving with Velocity v_s. As a Convention we have taken the direction from the observer to the Source as positive.

$$\boxed{\nu = \nu_0 \left(\frac{v}{v + v_s} \right)}$$

for a Source approating the observer

$$\boxed{\nu = \nu_0 \left(\frac{v}{v - v_s} \right)}$$



(ii) Observer Moving : Source Stationary

Let us consider a source of sound produ-
-cing a wave of angular frequency ω, Velocity v and time period T_0, to be at rest. while an observer initially at O_1 as moving with velocity v_s towards the source

$$\boxed{\nu = \nu_0\left(\frac{\nu + \nu_0}{\nu}\right)}$$

If the observer moves away from the source.

$$\boxed{\nu = \nu_0\left(\frac{\nu - \nu_0}{\nu}\right)}$$

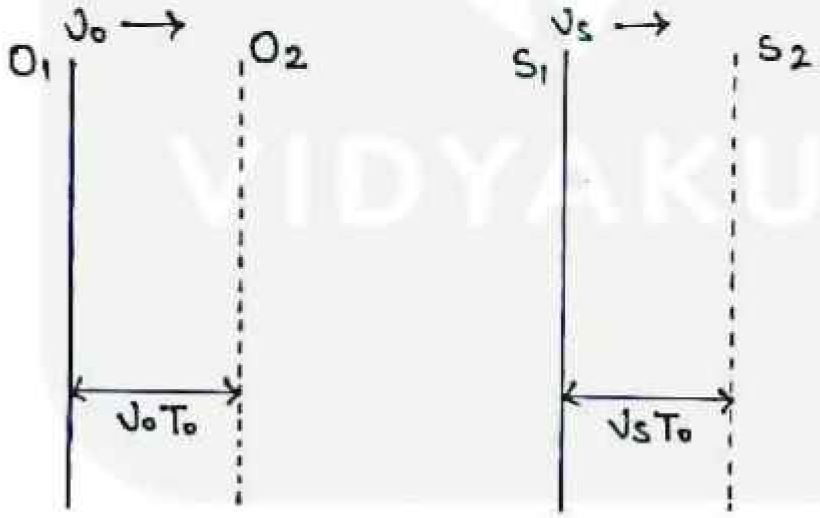
(iii) Both Source and Observer Moving

Let us consider that both the source and observer are moving the velocity ν_s and

υ_0 respectively. The source emits a wave of angular frequency ω, velocity v and time period T_0.

The observed frequency is given by

$$\boxed{\upsilon = \upsilon_0\left(\frac{v + \upsilon_0}{v + \upsilon_s}\right)}$$



Doppler effect when both the source and observer are moving with different velocities.