Chapter -14

Oscillations

In our daily life we come across various kinds of motions. The motion is repeated after a certain interval of time is called periodic motion. The study of Oscillation motion is basic to physics. In musical instruments, like the sitar, the guitar or the violin, We come across vibrating strings that produce pleasing sounds. The Vibratio-ns of air molecules make the propogation of Sound possible.

Periodic and Oscillatory Motions

Periodic Motions

Periodic Motion is defined as that motion which repeats itself after equal intervals of time. The interval of time is called the time period of periodic motion.

Ex- Rotational motion of Earth about its rotational axis and its time period is 24 hour.

Oscillatory Motions

Oscillatory or Vibratory motion is defined as a periodic and bounded motion of a body about a fixed point.

Ex- Motion of the pendulum of a wall Clock, motion of the bob of a simple pendulum displaced once from its mean position.

Period and Frequency

Period

Period is the smallest interval of time after which the motion is repeated. It is denoted by the symbol T.

Its SI unit is Second.

Frequency

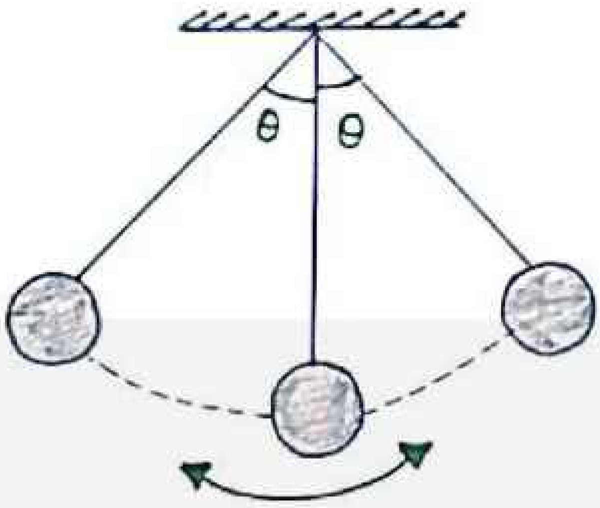
Frequency is defined as the number of Oscillations per unit time. It is the reciprocal of time period T. It is represented by the symbol f. Its SI unit is hertz (Hz).

The relation between f and T is

$$\boxed{f \propto \frac{1}{T}}$$

Displacement

The angle with the vertical as a function of time is the displacement variable. Consider a block attached to a spring, whose other end is fixed to a rigid wall. Here, it is convenient to measure displacement of the body from its equilibrium position.



Simple Harmonic Motion

Simple harmonic motion is a special type of periodic oscillatory motion in which

(i) The Particle Oscillates on a straight line.

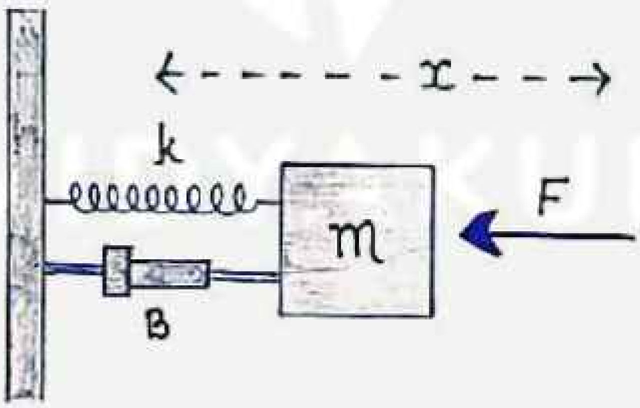
(ii) The acceleration of the particle is always directed towards a fixed point on the line.

(iii) The magnitude of acceleration is proportional to the displacement of the particle.

Now, Consider there is a Spring that is fixed at one end. When there is no force applied to it, it is at its equilibrium position. Now,

⇒ If we pull it outwards, there is a force exerted by the string that is directed towards the equilibrium position.

⇒ And If we push the spring inwards, there is a force exerted by the string towards the equilibrium position.



In each case, the force exerted by the spring is towards the equilibrium position, this force is called the restoring force.

Now, let the force be F and the displacement of the string from the equilibrium position be x.

Therefore, the restoring force will be $F = -kx$ (the negative sign indicates that the force is in the opposite direction.

Here, k = force constant

Its unit is N/m in S.I. system and dynes/cm in C.G.S. system.

Concept of Simple Harmonic Motion

Amplitude

The maximum displacement of a particle from its equilibrium position or mean position is its Amplitude and its direction is always away from the mean or equilibrium position.

Its S.I. unit is the meter, and the dimensions are $[L^1M^0T^0]$.

Period

The time taken by a particle to complete one oscillation is its period. Therefore, the period of S.H.M. is the least time after which the motion will repeat itself. Thus, the motion will repeat itself after nT, where, n is an integer.

Frequency

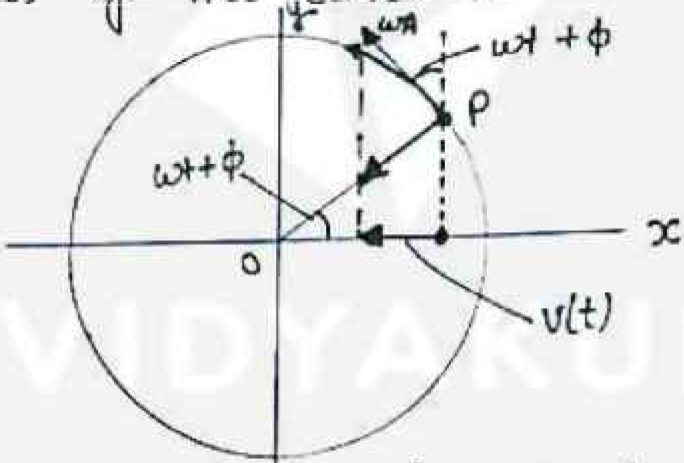
Frequency of S.H.M. is the number of oscillations that a particle performs per unit time. The S.I. unit of frequency is hetz or r.p.s. (rotations per second), and its dimensions are $[L^0 M^0 T^{-1}]$.

Phase

Phase of S.H.M. is its state of oscillations and the magnitude and direction of displacement of particles represent the phase.

Velocity and acceleration in Simple Harmonic Motion

The speed v of a particle in uniform Circular motion is, its angular speed ω times the radius of the circle A.



The direction of Velocity v at a time t is along the tangent to the circle at the point where the particle is located at that instant. From the geometry of the given figure, it is clear that the velocity of the projection particle P' at time t is

$$\boxed{v(t) = -\omega A \sin(\omega t + \phi)}$$

where the -ve sign shows that direction of v(t) is opposite to the +ve direction of x- axis.

The instantaneous acceleration of the projection of particle P' is then

$$\boxed{a(t) = -\omega^2 A \cos(\omega t + \phi)}$$

Maximum and minimum velocity

Velocity of a particle performing S.H.M. is given by

$$\boxed{V = \pm \omega \sqrt{a^2 - x^2}}$$

At mean position, $x = 0$. Therefore,

$$V = \pm \omega \sqrt{a^2 - 0^2} = \pm \omega \sqrt{a^2} = \pm a\omega$$

Therefore, at mean position, velocity of the particle performing SHM is maximum which is

$$V_{max} = \pm a\omega$$

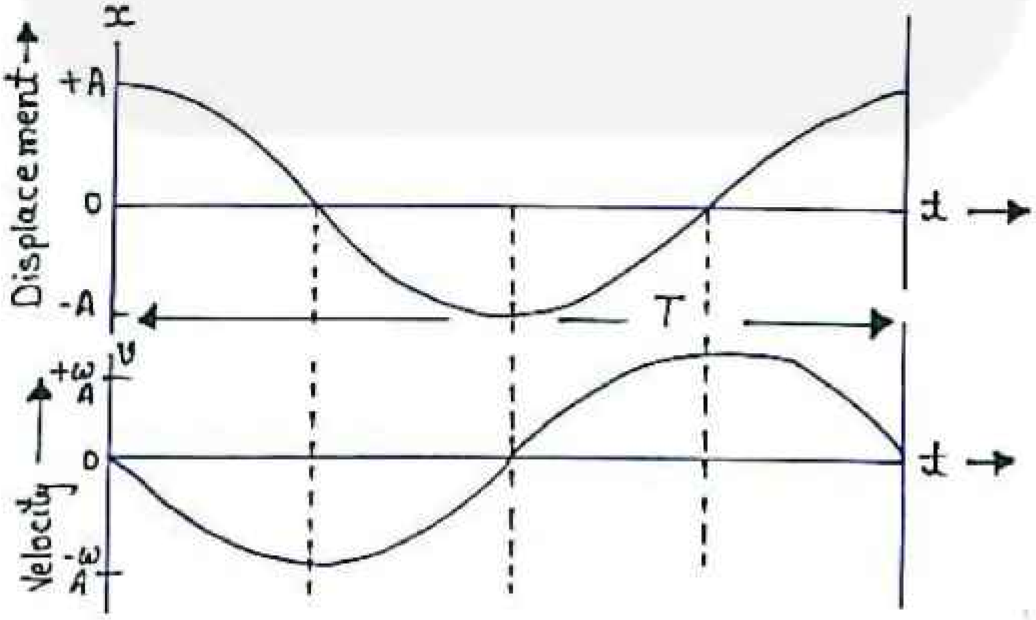
At extreme position, $x = \pm a$

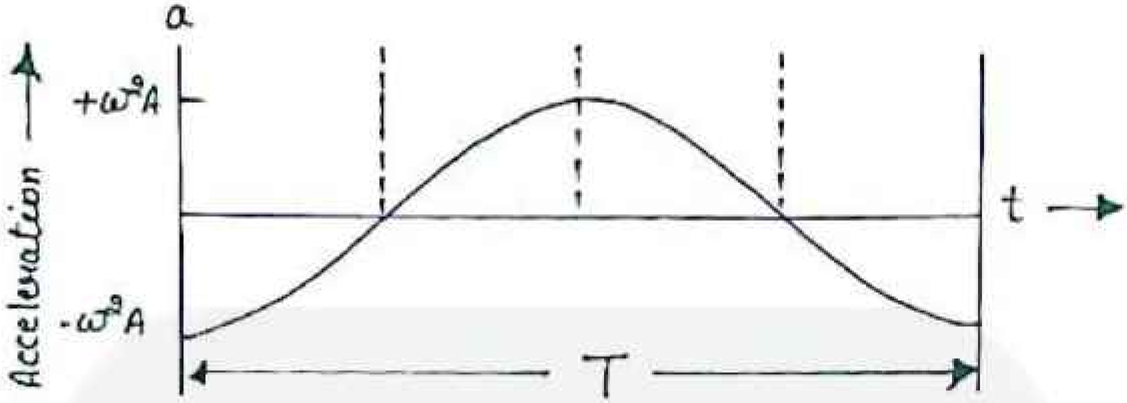
Therefore,

$$V = \pm \omega\sqrt{a^2 - a^2} = \omega \times 0 = 0$$

Therefore, at extreme position, velocity of the particle performing S.H.M. is minimum which is

$$V_{min} = 0$$





Force Law for Simple Harmonic Motion

using Newton's second Law of motion and the expression for acceleration of a particle undergoing SHM. The force acting on a particle of mass m in SHM is

$$\boxed{F(t) = ma}$$

$$= -m\omega^2 x(t)$$

i.e.

$$F(t) = -kx(t)$$

where

$$k = m\omega^2$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

Springs in Series

If two springs, having spring constant k_1 and k_2 are joined in series, the Spring Constant of the Combination is given by

$$\boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}}$$

Springs in Parallel

If two springs, having spring constant k_1 and k_2 are joined in parallel, the spring constant of the Combination is given by

$$\boxed{k = k_1 + k_2}$$

When one spring is attached to two masses m_1 and m_2, then

$$\boxed{M = \frac{m_1 m_2}{(m_1 + m_2)}}$$

Energy in Simple Harmonic Motion

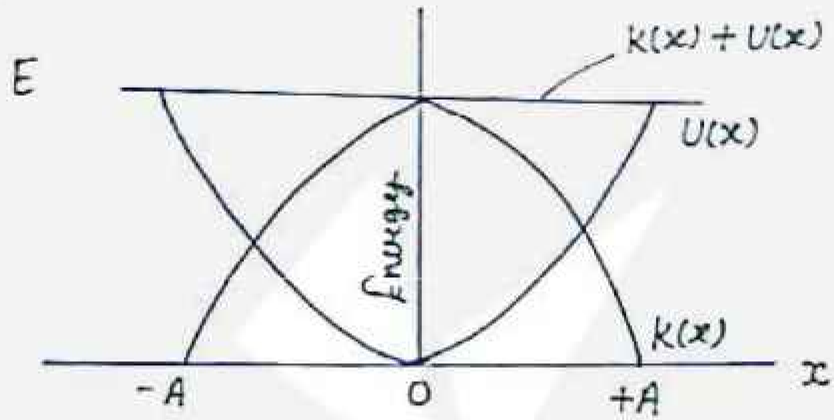
KE and PE of a particle in SHM vary between zero and their maximum values. The velocity of a particle executing SHM is zero at the extreme positions. So the kinetic energy (K) of such a particle is

$$\boxed{K = \frac{1}{2} m v^2}$$

$$K = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \phi)$$

$$K = \frac{1}{2} k A^2 \sin^2(\omega t + \phi)$$

This is also a periodic function of time, being zero when the displacement is maximum and minimum when the particle is at the mean position.



Spring force, $F = -kx$ is a conservative force, with associated potential energy.

$$\boxed{U = \frac{1}{2} k x^2}$$

So, the P.E. of a particle executing SHM is,

$$U = \frac{1}{2} k A^2 \cos^2 (\omega t + \phi)$$

So, PE of a particle executing SHM is

also periodic, with Period $\frac{T}{2}$, being zero at the mean position and Maximum at the extreme displacements.

The total energy E of the System is

$$E = U + K$$

$$E = \frac{1}{2} kA^2 \cos^2(\omega t + \phi) + \frac{1}{2} kA^2 \sin^2(\omega t + \phi)$$

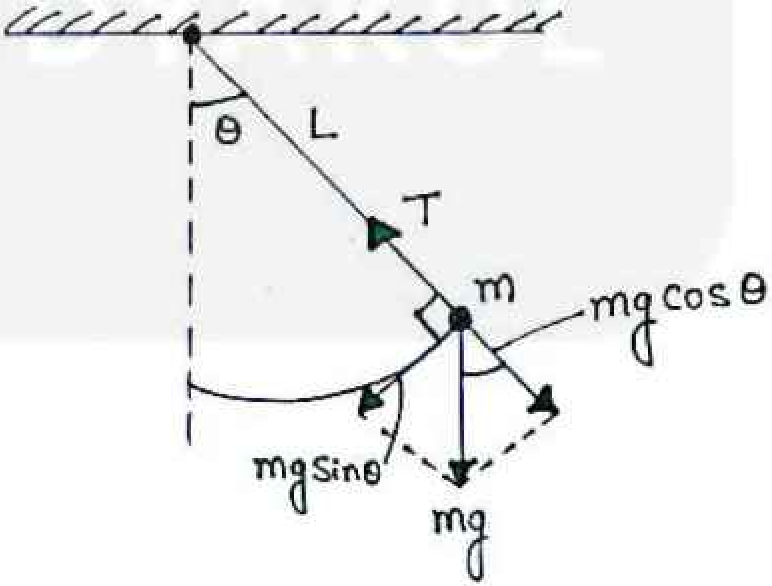
$$E = \frac{1}{2} kA^2 \left[\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right]$$

$$\boxed{E = \frac{1}{2} kA^2}$$

The total mechanical energy of a harmonic Oscillator is thus independent of time as expected for motion under any Conserva-tion force.

The Simple Pendulum

Make a Simple pendulum by Tying a piece of Stone to a long unstretchable thread, approximately 100 Cm long. Suspend it from a Suitable Support So that it is free to Oscillate. Displace the Stone to one Side by small distance and let it go. The Stone executes to and fro motion (periodic motion) with a period of about 2 Seconds.



Consider a Simple pendulum. A Small bob of mass m is tied to an inextensible massless string of length L. The other end of the string is fixed to a support on the ceiling. The bob oscillates in a plane about the vertical line through the support. Let θ be the angle which the string makes with the vertical. Here θ = 0 when the bob is at the mean position. The two forces acting on the bob are

1. The Tension T along the string
2. The vertical force due to gravity (mg).

The formula for time period of a simple pendulum is given by

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$