Chapter -10

Mechanical properties of fluids

Fluids are the sustances which can flow e.g. liquids and gasses. It does not possess definite shape.

When an object is submerged in a liquid at rest, the fluid exerts a force on its surface normally. It is called thrust of the liquid.

Pressure

The thrust experienced per unit area of the surface of a liquid at rest is called pressure.

$$\boxed{P = \frac{F}{A}}$$

It CGS System, unit of pressure is dyne cm^{-2}. In SI, unit of pressure is Nm^{-2}

or Pascal (Pa).

$$\boxed{1\,Pa = 1\,Nm^{-2}}$$

When a liquid is in equilibrium, the force acting on its surface is perpendicular everywhere. The pressure is the same at the same horizontal level.

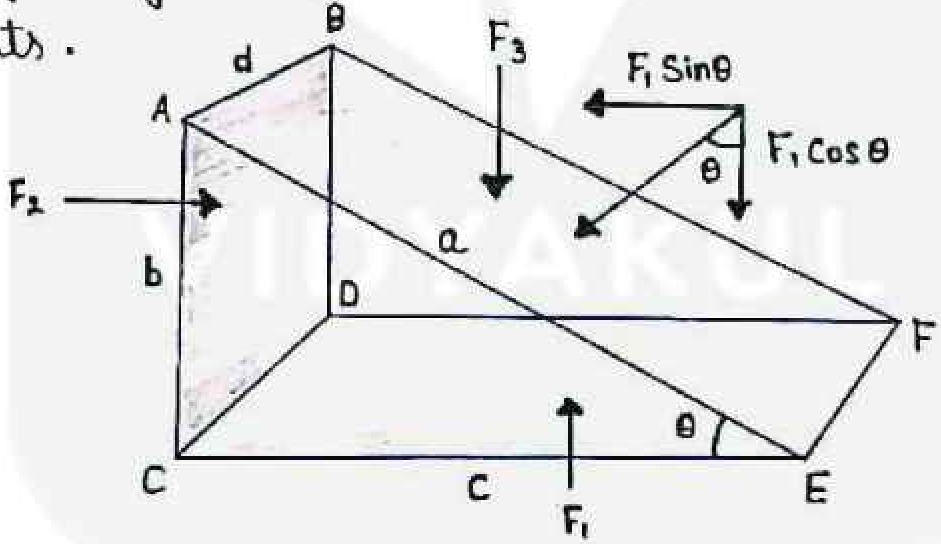
The pressure at any point in the liquid depends on the depth (h) below the surface, density of liquid and acceleration due to gravity.

Pascal's Law

According to Pascal's Law, the pressure applied to an enclosed liquid is trans-mitted undiminished to every portion of the liquid and the walls of the containing vessel.

$$\boxed{F = P \cdot A}$$

Consider an arbitrary right angled prismatic triangle in the liquid of density (ρ). The prismatic element is very small, every point is considered to be at the same depth as the liquid surface. Therefore, the effort of gravity is also the same at all these points.



Let ad, bd and cd be the area of the faces ABFE, ABDC and CDFE.

Let P_1, P_2 and P_3 be the pressure on the faces ABFE, ABDC and CDEF.

Pressure exerts a force which is normal to the surface. Let P_1 exert force F_1 on the surface ABFE, P_2 exert force F_2 on the surface ABDC and P_3 exert force F_3 on the surface CDFE.

Therefore, Force F_1, F_2 and F_3 is given as :

$$F_1 = P_1 \times \text{area of } ABFE = P_1 ad$$

$$F_2 = P_2 \times \text{area of } ABDC = P_2 bd$$

$$F_3 = P_3 \times \text{area of } CDFE = P_3 cd$$

Also,

$$\sin\theta = \frac{b}{a}$$

$$\cos\theta = \frac{c}{a}$$

The net force on the prism will be zero since the prism is in equilibrium.

$$F_1 \sin\theta = F_2$$

$$F_1 \cos\theta = F_3$$

$$P_1 ad \frac{b}{a} = P_2 bd \quad \text{—— (i)}$$

$$P_1 ad \frac{c}{a} = P_3 cd \quad \text{—— (ii)}$$

from equ. (i) and (ii)

$$P_1 = P_2 \text{ and } P_1 = P_3$$

$$\boxed{P_1 = P_2 = P_3}$$

Variation of pressure with Depth

Let's find the difference in pressures at two points, whose levels differ by a hight h in a fluid at rest.

Let P_1 and P_2 be the pressures at two points 1 and 2 inside a fluid. Point 1 is at a hight h above the point 2.

Imagine a fluid element in the shape of a Cylinder as shown. If A be the area of the top and the bottom of this cylinder, then

$$F_1 = P_1 A$$

$$F_2 = P_2 A$$

Since, the fluid remains at rest, therefore the force F_2, which acts upwards should balance the two downward forces. There are, the force F_1 exerted at the top of the cylinder, and the weight W of the fluid Confined within the cylinder.

$$\boxed{F_2 = F_1 + mg}$$

If ρ is the density of the fluid, then

$$F_2 = F_1 + \rho.(\text{Volume of the Cylinder}).g$$

$$P_2 A = P_1 A + \rho \cdot (A \cdot h) \cdot g$$

$$P_2 = P_1 + \rho g h$$

$$\boxed{P_2 - P_1 = \rho g h}$$

This result tells us that as we go deep down a liquid the pressure goes on increasing. This pressure depends only on the hight of liquid coloumn above the point.

Atmospheric Pressure and Gauge pressure

The Atmospheric pressure at a point is equal to the weight of a column of air of unit cross-sectional area extending from the point to the top of the atmosphere. Its value is 1.013×10^5

Pa at sea level.
Atmospheric pressure is measured using an instrument called barometer.

Units of Atmospheric Pressure

⇒ SI unit of pressure is Nm^{-2} or Pascal (Pa).

⇒ Atmosphere,

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} = 760 \text{ mm of Hg}$$

$$\boxed{1 \text{ torr} = 133 \text{ Pa}}$$

$$\boxed{1 \text{ mm of Hg} = 1 \text{ torr}}$$

$$\boxed{1 \text{ bar} = 10^5 \text{ Pa}}$$

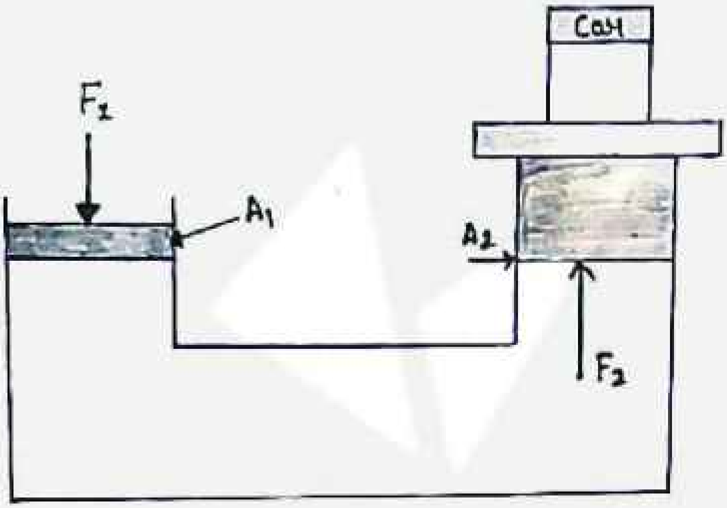
$$\boxed{1 \text{ millibar} = 100 \text{ Pa}}$$

Gauge Pressure

When we remove atmospheric pressure from total Pressure of any system then this remaining pressure is called "Gauge pressure". The excess pressure $P - P_a$, at depth h is called a gauge pressure at that point.

Hydraulic Machines

According to Pascal's Law of transmission, whenever external pressure is applied to any part of a liquid. This pressure gets distributed in all directions equally. A number of devices are based on this principle. The hydraulic lift is also an application of this Law. Consider a hydraulic lift as given in the figure below. The two pistons are seperated by a space filled with liquid. A piston of small cross section A_1 is used to exert a force F_1 directly on the liquid.



The pressure that is exerted in the column is given by $P = \frac{F_1}{A_1}$.

This is transmitted throughout the liquid, which results in the pressure being applied on the other piston. The area of the other piston is A_2, the force felt by this piston is given by,

$$P = \frac{F_2}{A_2}$$

$$\Rightarrow \quad \boxed{\frac{F_1}{A_1} = \frac{F_2}{A_2}}$$

$$\Rightarrow \quad F_2 = \frac{F_1 A_2}{A_1}$$

Notice that the applied force is increased by the factor of $\frac{A_2}{A_1}$. This property helps in hydraulic systems for lifting very heavy weights

Archimedes' Principle

When a body is partially or completely immersed in a liquid, it loses some of its weight. The loss in weight of the body in the liquid is equal to the weight of the liquid displaced by the immersed part of the body. The upward force excerted by the liquid displaced when a body is immersed is called buoyancy. Due to this, there is apparent loss in the weight experienced by the body.

Law of Floatation

A body floats in a liquid if weight of the liquid displaced by the immersed position of the body is equal to the weight of the body. When a body is immersed partially or wholly in a liquid, then the various forces acting on the body are

1. upward thrust (T) acting at the centre of buoyancy and whose magnitute is equal to the weight of the liquid displaced.

2. The weight of the body (W) which acts vertically downward through its centre of gravity.

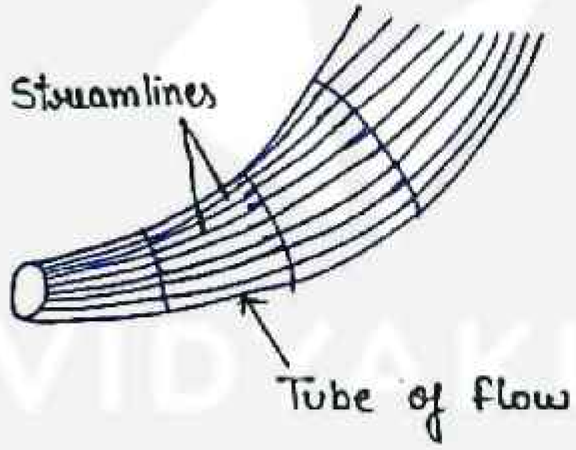
(a) When $W > T$, the body will sink in the liquid.

(b) When $W = T$, then the body will remain in equilibrium inside the liquid.

(c) when $W < T$, then the body will come upto the surface of the liquid.

Streamline flow

When a liquid flows such that each particle of the liquid passing a given point moves along the same path and has the same velocity as its predecessor had at that point, the flow is called stream lined or steady flow. The path followed by a fluid particle in steady flow is called streamline.



Equation of Continuity

According to this theorem, "For the stream line flow of an incompressible fluid through a pipe of varying cross-section, product of cross-section area and velocity of streamline flow (Av) remains constant throughout the flow.

$$\boxed{AV = \text{Constant}}$$

Bernoulli's Theorem

For an incompressible, non-viscous, irrotational liquid having streamlined flow, the sum of the pressure energy, kinetic energy and potential energy per unit mass is a constant i.e.,

$$\frac{P}{\rho} + \frac{v^2}{2} + gh = \text{Constant}$$

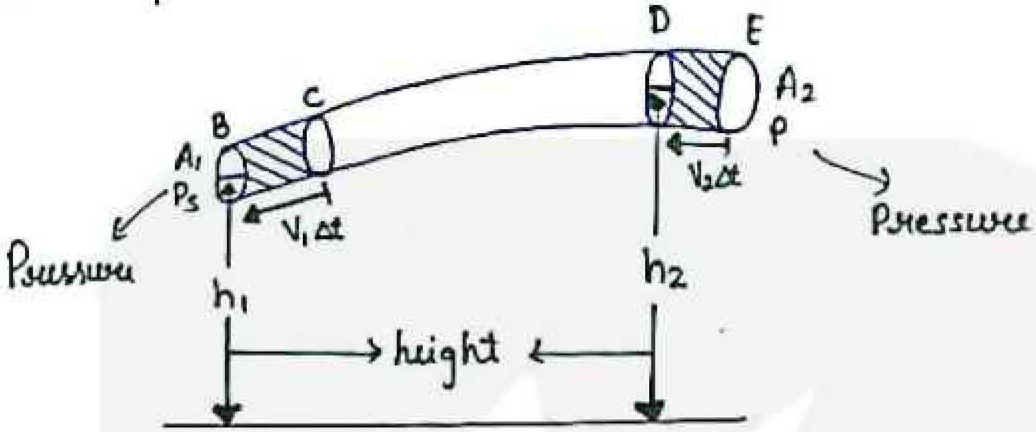
$$\boxed{\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{Constant}}$$

Deriving Bernoulli's Equation

Let the first region as BC and the Second region as DE. Now Consider the fluid was previously present in between B and D. However, this fluid will move in a minute interval of time (Δt).

If the speed of fluid at point B is V_1 and at point D is V_2. Therefore, if the fluid initially at B moves to C then the distance is $V_1 \Delta t$. However, $V_1 \Delta t$ is very small and it Constant across the Cross-section in the region BC.

Similarly, during the same interval of time Δt the fluid which was previously present in the point D is now at E. Thus, the distance covered is $V_2 \Delta t$. Pressures, P_1 and P_2, will act in the two regions, A_1 and A_2, there by binding the two parts.



Movement of fluid from $V_1\Delta t$ to $V_2\Delta t$ in Δt time interval.

Demostration of flow of an ideal fluid in pipe having Different cross-section

The Work done (W_1) on the fluid in the region BC. Work done

$$W_1 = P_1 A_1 (V_1 \Delta t) = P_1 \Delta V$$

the some volume of fluid will pass through BC and DE. Therefore, work done by the fluid on the right-hand Side of the pipe or DE region is

$$W_2 = P_2 A_2 (V_2 \Delta t) = P_2 \Delta V$$

Consider the work done on the fluids as $-P_2 \Delta V$. Total work done

$$\boxed{W_1 - W_2 = (P_1 - P_2)\Delta V}$$

The total work done help to Convert the gravitational potential energy and kinetic energy of the fluid.

The fluid density as ρ and the mass passing through the pipe as Δm in the Δt interval of time.

$$\Delta m = \rho A_1 V_1 \Delta t = \rho \Delta V$$

Now, the change in gravitational potential energy ΔU.

$$\Delta U = \rho g \Delta V (h_2 - h_1)$$

Similarly, the change in ΔK or kinetic energy

$$\Delta K = \left(\frac{1}{2}\right) \rho \Delta V (v_2^2 - v_1^2)$$

Applying work - energy theorem in the volume of the fluid.

$$(P_1 - P_2)\Delta V = \left(\frac{1}{2}\right) \rho \Delta V (v_2^2 - v_1^2) + \rho g \Delta V (h_2 - h_1)$$

$$(P_1 - P_2) = \left(\frac{1}{2}\right) \rho (v_2^2 - v_1^2) + \rho g (h_2 - h_1)$$

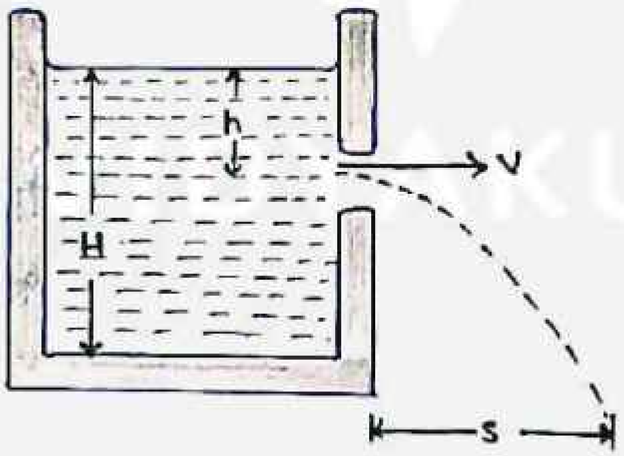
$$P_1 + \left(\frac{1}{2}\right) \rho v_1^2 + \rho g h_1 = P_2 + \left(\frac{1}{2}\right) \rho v_2^2 + \rho g h_2$$

General equation

$$\boxed{P + \left(\frac{1}{2}\right) \rho v^2 + \rho g h = \text{Constant}}$$

Speed of Efflux : Torricelli's Law

Velocity of efflux (the velocity with which the liquid flows out of a orifice or narrow hole) is equal to the velocity acquired by a freely falling body through the same vertical distance equal to the depth of orifice below the free surface of liquid.



$$\boxed{\text{Velocity of efflux, } v = \sqrt{2gh}}$$

where

h = depth of orifice below the free surface of liquid

$$t = \sqrt{\frac{2(H-h)}{g}}$$

$$\boxed{\text{Horizontal range, } S = \sqrt{4h(H-h)}}$$

where

H = hight of liquid Column

If the hole is at the bottom of the tank, then time required to make the tank empty is,

$$t = \frac{A}{A_0}\sqrt{\frac{2H}{g}}$$

where A = area of Container

A_0 = area of Orifice

Volume of liquid Coming out from the Orifice per second.

$$VA_0 = A_0\sqrt{2gh}$$

$$\boxed{V = \sqrt{2gh}}$$

Viscosity

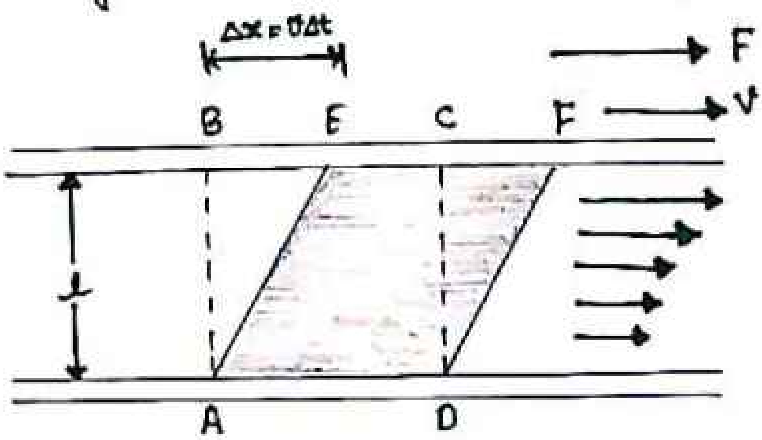
Viscosity is the property of the fluid (liquid or gas) by virtue of which an internal frictional force comes into play when the fluid is in motion in the form of layers having relative motion. It opposes the relative motion of the different layers.

Viscosity is also called as fluid friction.

The viscous force directly depends on the area of the layer and the velocity gradient.

$$\boxed{F = -\eta A \frac{dv}{dx}}$$

(-ve sign shows the opposing nature)



Coefficient of Viscosity

Coefficient of Viscosity of a liquid is equal to the tangential force required to maintain a unit velocity gradient between two parallel layers of liquid each of area Unity.

$$\boxed{\eta = \frac{F}{A\left(\frac{dv}{dx}\right)}}$$

The SI unit of Coefficient of Viscosity is poiseuille (PI) or Pa - s or $Nm^{-2}s$ or $kg\ m^{-1}s^{-1}$. Dimensional formula of η is $[ML^{-1}T^{-1}]$.

Reynold's Number

Reynold number Re is a dimension less number whose value gives an approximate idea whether the flow of a fluid will be streamline or turbulent. It is given by

$$\boxed{R_e = \frac{\rho v d}{\eta}}$$

where ρ = density of fluid flowing with a speed v, d stands for the diameter of the pipe and η is the viscosity of the fluid. Value of R_e remains same in any systems of units.

It is observed that flow is streamline or laminar for $R_e <= 1000$ and the flow is turbulent for $R_e >= 2000$. The flow becomes unsteady for R_e between 1000 and 2000. The critical value of R_e, at which turbulence sets, is same for the geometrically similar flows.

R_e may also be expressed as the ratio of inertia force (force due to inertia i.e., mass of moving fluid or due to inertia of obstacle in its path) to viscous force i.e.,

$$R_e = \frac{\rho A v^2}{\left(\frac{\eta A v}{d}\right)}$$

Surface Tension

It is the property of the liquid by virtue of which the free surface of liquid at rest tends to have minimum area and as such it behaves as a stretched elastic membrane.

The force acting per unit length of line drawn on the liquid surface and normal to it parallel to the surface is called the force of surface tension.

The SI unit of surface tension is Nm^{-1} and its dimensional formula is $[MT^{-2}]$.

Surface energy

Energy possessed by the surface of the liquid is called surface energy.
Change in surface energy is the product

of surface tension and change in surface area under constant temperature.

The height to which water rises in a capillary tube of radius r is given by

$$\boxed{h = \frac{2T\cos\theta}{r\rho g}}$$

where

T = Surface Tension of the liquid

θ = angle of contact

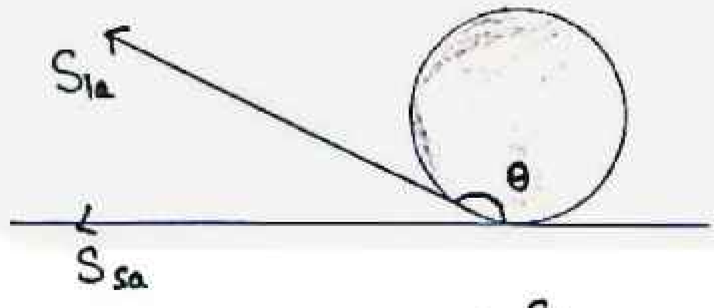
Due to surface tension there is excess pressure on the concave side of a surface film of a liquid over.

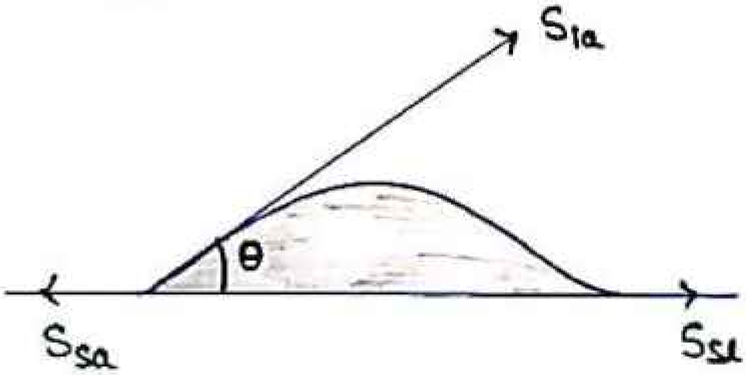
The convex side and is equal to $\frac{2T}{r}$. For a soap bubble the excess pressure is $\frac{4T}{r}$. where r = radius of the surface.

Angle of Contact

The angle which the tangent to the liquid surface at the point of contact makes with the solid surface inside the liquid is called angle of contact.

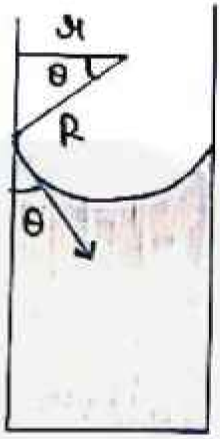
Intermolecular force amongst molecules of the same material is called the force of cohesion. However, force amongst molecules of different materials is called the force of adhesion.





Capillary Rise

The pressure due to the liquid (water) column of height h must be equal to the pressure difference $\frac{2T}{R}$ due to the concavity.



$$\boxed{h\rho g = \frac{2T}{R}}$$

where ρ = density of the liquid

g = acceleration due to gravity

Let r be the radius of the capillary tube and θ be the angle of contact of the liquid.

Then radius of curvature R of the meniscus is given by

$$R = \frac{r}{\cos\theta}$$

$$h\rho g = \frac{2T\cos\theta}{r}$$

$$\boxed{h = \frac{2T\cos\theta}{r\rho g}}$$

The Given expression for capillary rise for a liquid. Narrower the tube, the greater is the height to which the liquid rises (or falls).